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Phase and amplitude control of switching from positive to negative dispersion in superconducting quantum circuits

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ABSTRACT

We study the absorption and dispersion properties of the three-level Δ-type superconducting fluxonium circuit, in which each atomic transition is respectively driven by a coherent classical field. The results show that the absorption and dispersion spectra depend strongly on the relative phase and intensities of the applied fields. When the relative phase is changed from $\pi/2$ to $3\pi/2$, the switching from positive to negative dispersion arises, which can also be obtained via adjusting the relative intensities of the classical fields. Our scheme shows that the dispersion switching effect could be achievable for microwave pulse interacting with the superconducting fluxonium qubit.

1. Introduction

Quantum coherence and interference effects are currently attracting considerable interest in quantum optics and atomic physics owing to their interesting phenomena [1–8]. It is found that strong-field driving plays a crucial role in quantum coherence effects. In particular, the phase-controlled optical phenomena have attracted much attention in recent years [9–23]. For example, the phase-dependent electromagnetically induced transparency (EIT) has been investigated in the atomic system [9–11]. The coherent control of spontaneous emission can be effectively carried out via the laser phases [12,13]. The schemes for atom localization have been presented via phase control of absorption spectra

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[14–16]. The light pulse with subluminal and superluminal group velocity can be controlled by the relative phase of the applied fields [17–19]. The recent researches show that the phase of the laser could be applied to manipulate quantum entanglement [20,21]. It is noted that the schemes previously presented are dedicated to the phase-modulated coherent effects in the atomic system.

On the other hand, superconducting quantum circuits as solid state devices become the important candidates for the fundamental studies and practical applications in quantum optics and quantum information [24–40]. This is because that these devices have the following distinctive features. First, the superconducting qubits have the tunability and controllability of parameter and experimental fabrication, which can be designed on demand. Second, the interaction could occur between a single superconducting qubit and a microwave field in one dimension, which can overcome the spatial-mode mismatch between incident and scattered waves. Third, the characteristic frequencies of the qubit belong to the microwave frequency range and this will be useful for the low-noise microwave information processing. As a consequence, many interesting optical phenomena have been demonstrated in such a device, such as resonance fluorescence [25], EIT [26,27], Autler–Townes splitting [28], coherent population trapping [29], light storage [30], Kerr effect [31], optical mixing [32] and so on. More recently, the superconducting qubits have been proposed to implement quantum information processing [41,34–36].

In this paper, we discuss the feasibility of dispersion switching with nearly-vanishing absorption in a single driven three-level Δ-type superconducting fluxonium qubit. It is shown that the absorption and dispersion properties can be effectively controlled by the relative phase and intensities of the external fields. When the relative phase is changed from $\pi/2$ to $3\pi/2$, the dispersion can be switched from positive to negative with the nearly-vanishing absorption. Furthermore, the intensities of the external fields could also be used to control the dispersion and absorption behavior. The scheme that we present displays the microwave pulse propagation and microwave coherent effects in the superconducting quantum circuit.

2. Physical model and equation

We first consider a single superconducting fluxonium qubit (shown in Fig. 1(a)) and write its Hamiltonian as [37–40]

$$H_f = 4E_C N^2 - E_J \cos\left(\phi - 2\pi \Phi_{\text{ext}}/\Phi_0\right) + E_L \phi^2/2. \tag{1}$$

Here the operator $N = Q/(2e)$ denotes the charge imbalance across the small junction, in units of the Cooper pair box, and $\phi = 2\pi\Phi/\Phi_0$ describes the phase difference. The coefficients E_C, E_J, E_L represent the energies with $E_C = e^2/\left(2C_J\right), E_J = (\Phi_0/2\pi)^2/\left(L_J\right), E_L = (\Phi_0/2\pi)^2/(L_A)$, respectively. The parameter Φ_{ext} is the external magnetic flux and Φ_0 is the flux quantum $\Phi_0 = h/(2e)$. Using the method of fluxonium Hamiltonian diagonalization [39], the Hamiltonian H_f takes the form $H_f = \sum_l \hbar\omega_l |l\rangle\langle l|$ with the spectroscopy frequencies ω_l, which means the fluxonium qubit has the quantized energy levels as same as those of the atom. Meanwhile, the amplitudes of photon-induced transitions between different energy levels are then determined by the charge matrix elements $N_{jk} = \langle j| N |k\rangle$ with j and k being the eigenstates, which corresponds to the case of the fluxonium qubit being capacitively coupled to a transmission line.

For studying the coherent optics effects, here we focus on the lowest three energy levels of the fluxonium qubit and we present its transition frequencies and the charge matrix elements (in units of $2e$) vs. the flux bias Φ_{ext}/Φ_0 (shown in Fig. 1(b,c)), with the parameters being from Refs. [37,38]. Obviously, all three matrix elements have comparable values when the flux bias is away from $\Phi_{\text{ext}}/\Phi_0 = 0$ and ± 0.5, and the Δ-type three-level structure is generated, which is absent in the atomic system. Here we use the interactions of such a system with the external applied fields to obtain the interesting and useful effects. For the fluxonium, the three lowest states are respectively given by $|1\rangle$, $|2\rangle$ and $|3\rangle$, and a coherent electromagnetic field is applied on each atomic transition. In the electronic-dipole and rotating wave approximation, the total Hamiltonian is expressed as ($\hbar = 1$)

$$H = \sum_{l=1-3} \omega_l |l\rangle\langle l| - \frac{1}{2}\left(\sum_{j>k} \Omega_{jk} e^{i\left(\nu_{jk}t + \phi_{jk}\right)} |j\rangle\langle k| + \text{H.c.}\right). \tag{2}$$