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Implicit and explicit renormalization: Two complementary views of effective interactions

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We analyze quantitatively the interplay between explicit and implicit renormalization in Nuclear Physics. By explicit renormalization we mean to integrate out higher energy modes below a given cutoff scale using the similarity renormalization group (SRG) with a block-diagonal evolution generator, which separates the total Hilbert-space into a model space and its complementary. In the implicit renormalization we impose given conditions at low energies for a cutoff theory. In both cases we compare the outcoming effective interactions as functions of the cutoff scale. We carry out a comprehensive analysis of a toy-model which captures the main features of the nucleon–nucleon (*NN*) *S*-wave interaction at low energies. We find a wide energy region where both approaches overlap. This amounts to a great simplification in the determination of the effective interaction. Actually, the outcoming scales are within the expected ones relevant for the physics of light nuclei.

1. Introduction

The idea of renormalization group from a Wilsonian point of view is quite intuitive and appealing [1]. A truncated Hilbert space is considered below some given maximal energy where the relevant

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physical degrees of freedom are taken into account explicitly. All states above that maximal energy are integrated out and contribute to the structure of operators and their couplings in the reduced Hilbert state via scale-dependent effective interactions. The renormalization group equations arise from the requirement that physical results ought to be independent on the chosen numerical maximal energy value. While one may identify a fundamental underlying theory with the corresponding elementary degrees of freedom, the so-called *ab initio* calculations may not necessarily be the most efficient way to pose the quantum mechanical many-body problem of composite and extended interacting constituent particles. Actually, for a self-bound system its compositeness vs the elementary character depends on the shortest de Broglie wavelength involved in the physical process under consideration as compared to the typical length scales characterizing the interaction among constituents. For *known* interactions the Wilsonian renormalization group approach proves a convenient computational strategy to tackle the many-body problem.

In Nuclear Physics the interaction among nucleons is *unknown* fundamentally and precisely except at long distances where one-pion Exchange (OPE) dominates. At shorter distances the interaction may be constrained from fits to nucleon–nucleon (*NN*) scattering data up to a given maximum energy and with a given accuracy (see Refs. [2–5] for the most recent upgrade in the elastic regime for np and pp data). Thus, the particular status of the nuclear force makes renormalization group methods an ideal tool to address the problem of nuclear binding.

In the case of atomic nuclei glued together by the *NN* force the most troublesome issue for nuclear structure calculations is the appearance of an inner large core of about $a_c = 0.5$–0.6 fm which becomes visible for *NN* scattering at pion production threshold [6] (see however Ref. [7] for an alternative interpretation). This distinct feature generates a strong short-range repulsion which complicates enormously the solution of the multinucleon problem limiting the maximal number of nucleons in *ab initio* calculations [8]. On the other hand, because the net effect of the core is to prevent particles to come too close in the nucleus ground-state the net contribution to the binding-energy stemming from distances smaller than the core is tiny. From this point of view one may equally assume weakly interacting particles at short distances, thus under these circumstances the core may be replaced by a suitable soft-core short-distance interaction which keeps invariant the scattering information.

With this perspective in mind, the idea of effective interactions has been developed after the early proposals of Goldstone [9], Moshinsky [10] and Skyrme [11] and Moszkowski and Scott [12] as a way to cut the gordian knot of the Nuclear Many-Body Problem represented by strong short-range repulsion. This allowed to take advantage of the much simpler mean field framework based on those effective interactions [13] (for a review see e.g. [14]). The main problem of the effective interaction approach is both the proliferation of independent parameters as well as their huge numerical diversity (see e.g. the recent compilation of parameters [15]). This reflects both the lack of a unambiguous link to the fundamental two-body interaction as well as the quite disparate finite nuclei and nuclear matter observables which have been used to fix the effective Hamiltonian parameters. An effort has been made [16] (see e.g. Ref. [17] for a similar setup and [18,19] for alternative views) in order to understand the origin of the two-body effective interactions from free space *NN* scattering without invoking finite nuclei nor nuclear matter properties. This point of view corresponds to what will be called here as *implicit* renormalization.

These somewhat intuitive considerations have been made more precise by a novel re-interpretation of the Nuclear Many-Body Problem from the Wilsonian renormalization group point of view. The novel insight, dubbed as $V_{\text{low k}}$, is to provide an alternative approach to the determination of effective interactions *directly* from the *NN bare* potentials fitted to the scattering data [20–23] (for reviews see e.g. [24–27] and references therein) and their characterization as finite cutoff counterterms [28]. This point of view corresponds to what will be called here as the *explicit* renormalization. The basis of the whole framework has been to recognize the relevance of choosing the proper physical scale resolution in the formulation of the problem. This amounts to a great simplification since at the relevant scales the many-body problem is posed in terms of effective degrees of freedom and hence the interaction decreases and softens. Thus, a mean field solution can be used as a reliable zeroth order approximation, from where corrections can perturbatively be computed. Moreover, when the maximum energy is taken at about pion production threshold or below, the $V_{\text{low k}}$ interaction does not