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A hybrid approach for quantizing complicated motion of a charged particle in time-varying magnetic field

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ABSTRACT

Quantum characteristics of a charged particle subjected to a singular oscillator potential under an external magnetic field is investigated via SU(1,1) Lie algebraic approach together with the invariant operator and the unitary transformation methods. The system we managed is somewhat complicated since we considered not only the time-variation of the effective mass of the system but also the dependence of the external magnetic field on time in an arbitrary fashion. In this case, the system is a kind of time-dependent Hamiltonian systems which require more delicate treatment when we study it. The complete wave functions are obtained without relying on the methods of perturbation and/or approximation, and the global phases of the system are identified. To promote the understanding of our development, we applied it to a particular case, assuming that the effective mass slowly varies with time under a time-dependent magnetic field.

1. Introduction

The motion of a charged particle subjected to a time-dependent singular oscillator potential, as well as standard oscillator potential, in a time-dependent magnetic field attracted considerable attention

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so far due to its rich applications in actual dynamical systems. For instance, Dittrich et al. quantized two-dimensional motion of an electron whose effective mass is constant [1]. One of our authors also studied the similar problem but with the time-dependent effective mass [2]. If charged particles such as electrons or holes interact with environment or excitations such as pressure, temperature, stress and so on, their effective mass may be more or less modified [3–5]. In particular, if the external field depends on time in an arbitrary fashion, the effective mass of electrons in a heterojunction varies randomly on account of the effects of fluctuation of various excitations in the system [6]. The importance of this research relevant to these phenomena manifest from the fact that many quantum characteristics of nano devices are associated with the time behavior of electrons in it. According to the trend of miniaturization of electronic devices and the recent advance of technology for nanofabrication, the dominance of quantum features in (nano) devices gradually become distinct. If the dimension of elements in a semiconductor chip reaches a characteristic one known as the Fermi wave length, the classical mechanics cannot be available for describing physical properties of the system. Meanwhile, the miniaturization of electronic devices to a nanoscale accompanies quantum effects conspicuously and, as a consequence, our study of them become more and more crucial for the development of nanoscience.

As is well known, if some parameters of the system such as mass vary with time, the system is described by a time-dependent Hamiltonian. Among many possible methods useful for treating time-dependent Hamiltonian systems (TDHSs), we consider a hybrid approach with the choice of several mathematical techniques. So to say, invariant operator method and SU(1,1) Lie algebraic approach with a suitable unitary transformation will be employed here in order to study quantum problem of the charged particle with time-variable effective mass subjected by two-dimensional singular oscillator potential under time-varying magnetic field. The applicability of SU(1,1) Lie algebra in investigating dynamical problems of a TDHS is well known in the literature [7–11]. The strong point of this algebra against others is that it allows us to demonstrate the similarity of quantum mechanical behavior of TDHSs with the corresponding classical one [11]. For details of other mathematical techniques for treating TDHS, such as invariant operator method, unitary transformation method and others in quantum physics and chemistry, you can refer to Refs. [12–21].

In general, quantum solutions of TDHSs are derivable in terms of solutions of some classical equations of motion [2]. We will also introduce a solution of some classical equation of motion when developing our quantum theory of the system. Then, the quantum problem associated with time-dependent Hamiltonian can be solved without any approximation in so far as the exact solutions of the classical equation of motion are attainable. This is a fascinating idea if we recall that quantum solutions of lots of mechanical systems are derived relying on perturbation or approximation method up to now due to their complexity of mathematical structure [22–24]. In many cases, this enables one to show the similarity between quantum and classical behaviors of TDHS, as has done by Choi [25] for an optical system.

2. Survey of quantum Hamiltonian dynamics

The dynamical system of our interest is a charged particle with variable effective mass that follow a singular oscillator motion in two dimension in a variable magnetic field. Usually, singular oscillator is subjected to a harmonic plus inverse harmonic potential. Although many authors assume that the change of mass or other parameters in time is periodic in order to simplify the problem [26,27], our research that will be performed here will also encompass the case of the random change of mass. The Hamiltonian is dependent on time and known to be [28]

$$H(x, y, t) = \frac{[P - qA(t)]^2}{2M(t)} + \frac{1}{2}M(t)\varpi^2(t)(x^2 + y^2) + \frac{\mathbf{c}}{2M(t)}(x^2 + y^2)^{-\mathbf{1}}, \tag{1}$$

where $M(t)$ is a time-dependent effective mass, $\varpi(t)$ is a time-dependent frequency, $\mathbf{c}$ is a constant, and $\vec{P} = p_x \vec{i} + p_y \vec{j}$. Note that $[P - qA(t)]^2$ can be simplified by choosing an appropriate symmetric