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Torsion and noninertial effects on a nonrelativistic Dirac particle

K. Bakke *

Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, 58051-970, João Pessoa, PB, Brazil

HIGHLIGHTS

- Torsion effects on a spin-1/2 particle in a noninertial reference frame.
- Fermi–Walker reference frame in the cosmic dislocation spacetime background.
- Torsion and noninertial effects on the confinement to a hard-wall confining potential.

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ABSTRACT

We investigate torsion and noninertial effects on a spin-1/2 quantum particle in the nonrelativistic limit of the Dirac equation. We consider the cosmic dislocation spacetime as a background and show that a rotating system of reference can be used out to distances which depend on the parameter related to the torsion of the defect. Therefore, we analyse torsion effects on the spectrum of energy of a nonrelativistic Dirac particle confined to a hard-wall potential in a Fermi–Walker reference frame.

1. Introduction

In recent decades, a great deal of works has studied the influence of torsion on several physical systems [1–11]. The interaction between fermions and torsion and possible physical effects have been discussed in Refs. [4–6]. In crystalline solids, torsion has been studied in the continuum picture of defects by using the differential geometry in order to describe the strain and the stress induced by the defect in an elastic medium [12–17]. The study of the influence of torsion on quantum systems

* Tel.: +55 83 8801 5202.
E-mail addresses: kbakke@fisica.ufpb.br, kbakkef@gmail.com.

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has been extended to the electronic properties of graphene sheets [18], Berry's phase [19], quantum scattering [20], Landau levels for a nonrelativistic scalar particle [21] and holonomic quantum computation [22]. The influence of torsion on a two-dimensional quantum ring has been discussed in Ref. [23] and on a quantum dot in Ref. [24].

In this paper, we discuss torsion effects on the spectrum of energy of a nonrelativistic spin-1/2 particle confined to a hard-wall confining potential in a noninertial reference frame by considering the cosmic dislocation spacetime as a background and by showing that a rotating system of reference can be used out to distances which depend on the parameter related to the torsion of the defect. Studies of noninertial effects have discovered quantum effects related to geometric phases [25–27] and the coupling between the angular momentum and the angular velocity of the rotating frame [28–30]. In other quantum systems, the study of noninertial effects have been extended to the weak field approximation [31], the influence of Lorentz transformations [32], scalar fields [33], Dirac fields [34], persistent currents in quantum rings [35], spin currents [36] and rotational and gravitational effects in quantum interference [37–39]. An interesting discussion made in Ref. [40] is by making a coordinate transformation (in the Minkowski spacetime) from a system at rest to a uniformly rotating frame, then, the line element of the Minkowski spacetime is not well-defined at large distances. This means that the coordinate system becomes singular at large distances, which is associated with the velocity of the particle would be greater than the velocity of the light. In recent years, the behaviour of external fields and the wave function of a neutral particle have been analysed under the influence of noninertial effects and the presence or absence of curvature [41]. In Refs. [42,43], it has been shown that noninertial effects can provide a confinement of a neutral particle interacting with external fields analogous to a two-dimensional quantum dot. Thereby, our focus is to analyse torsion and noninertial effects and fill a lack in the study of the quantum dynamics of a nonrelativistic Dirac particle.

This paper is structured as follows: in Section 2, we study the nonrelativistic quantum dynamics of a spin-1/2 particle in the Fermi–Walker reference frame and in the presence of torsion. Then, we show that a rotating system of reference can be used out to distances which depend on the parameter related to the torsion of the defect and use the coordinate singularity to impose where the wave function must vanish, which allows us to obtain bound states analogous to the confinement of a spin-1/2 quantum particle to a hard-wall confining potential; in Section 3, we present our conclusions.

2. Torsion effects on the nonrelativistic quantum dynamics of a Dirac particle in a noninertial frame

In this section, we study the confinement of a nonrelativistic Dirac particle to a hard-wall confining potential under the influence of noninertial effects and torsion. We present the mathematical tools to describe spinors under the influence of torsion and noninertial effects, when we consider the local reference frame of the observers is a Fermi–Walker reference frame. In this paper, we work with the units $\hbar = c = 1$. We start by writing the line element of the cosmic dislocation spacetime [9,10,44] (in the rest frame of the observers):

$$ds^2 = -d\mathcal{T}^2 + d\mathcal{R}^2 + \mathcal{R}^2 d\Phi^2 + \left(d\mathcal{Z} + \zeta\, d\Phi\right)^2 . \quad (1)$$

The parameter ζ is a constant, and it is related to the torsion of the defect. From the crystallography language, the parameter ζ is related to the Burgers vector $\vec{b} = b\,\hat{z}$, where $\zeta = \frac{b}{2\pi}$ [7,9,44].

In the following, we consider a coordinate transformation given by $\mathcal{T} = t$, $\mathcal{R} = \rho$, $\Phi = \varphi + \omega t$, and $\mathcal{Z} = z$, where ω is the constant angular velocity of the rotating frame. Thus, the line element (1) becomes

$$ds^2 = -\left(1 - \omega^2\rho^2 - \zeta^2\omega^2\right) dt^2 + \left(2\omega\rho^2 + 2\zeta^2\omega\right) d\varphi\, dt + d\rho^2 + \rho^2 d\varphi^2 + 2\zeta\omega\, dz\, dt + \left(dz + \zeta\, d\varphi\right)^2 . \quad (2)$$

We can note that the line element (2) is defined for values of the radial coordinate inside the range:

$$0 < \rho < \frac{\sqrt{1 - \zeta^2\omega^2}}{\omega}. \quad (3)$$