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Scattering and bound states of fermions in a mixed vector–scalar smooth step potential

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HIGHLIGHTS

- Scattering and bound states of fermions in a kink-like potential.
- No pair production despite the high localization.
- No bounded solution under exact spin and pseudospin symmetries.

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ABSTRACT

The scattering of a fermion in the background of a smooth step potential is considered with a general mixing of vector and scalar Lorentz structures with the scalar coupling stronger than or equal to the vector coupling. Charge-conjugation and chiral-conjugation transformations are discussed and it is shown that a finite set of intrinsically relativistic bound-state solutions appears as poles of the transmission amplitude. It is also shown that those bound solutions disappear asymptotically as one approaches the conditions for the realization of the so-called spin and pseudospin symmetries in a four-dimensional space–time.

1. Introduction

The solutions of the Dirac equation with vector and scalar potentials can be classified according to an SU(2) symmetry group when the difference between the potentials, or their sum, is a constant.

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The near realization of these symmetries may explain degeneracies in some heavy meson spectra (spin symmetry) [1,2] or in single-particle energy levels in nuclei (pseudospin symmetry) [2,3]. When these symmetries are realized, the energy spectrum does not depend on the spinorial structure, being identical to the spectrum of a spinless particle [4]. In fact, there has been a continuous interest for solving the Dirac equations in the four-dimensional space–time as well as in lower dimensions for a variety of potentials and couplings. A few recent works have been devoted to the investigation of the solutions of the Dirac equation by assuming that the vector potential has the same magnitude as the scalar potential [5–7] whereas other works take a more general mixing [8–11].

In a recent work the scattering a fermion in the background of a sign potential has been considered with a general mixing of vector and scalar Lorentz structures with the scalar coupling stronger than or equal to the vector coupling [11]. It was shown that a special unitary transformation preserving the form of the current decouples the upper and lower components of the Dirac spinor. Then the scattering problem was assessed under a Sturm–Liouville perspective. Nevertheless, an isolated solution from the Sturm–Liouville perspective is present. It was shown that, when the magnitude of the scalar coupling exceeds the vector coupling, the fermion under a strong potential can be trapped in a highly localized region without manifestation of Klein's paradox. It was also shown that this curious lonely bound-state solution disappears asymptotically as one approaches the conditions for the realization of "spin and pseudospin symmetries".

The purpose of the present paper is to generalize the previous work to a smoothed out form of the sign potential. We consider a smooth step potential behaving as $V(x) \sim \tanh \gamma x$. This form for the potential, termed kink-like potential just because it approaches a nonzero constant value as $x \to +\infty$ and $V(-\infty) = -V(+\infty)$, has already been considered in the literature in nonrelativistic [12] and relativistic [13–15] contexts. The satisfactory completion of this task has been alleviated by the use of tabulated properties of the hypergeometric function. A peculiar feature of this potential is the absence of bound states in a nonrelativistic approach because it gives rise to an ubiquitous repulsive potential. Our problem is mapped into an exactly solvable Sturm–Liouville problem of a Schrödinger-like equation with an effective Rosen–Morse potential which has been applied in discussing polyatomic molecular vibrational states [16]. The scattering problem is assessed and the complex poles of the transmission amplitude are identified. In that process, the problem of solving a differential equation for the eigenenergies corresponding to bound-state solutions is transmuted into the solutions of a second-degree algebraic equation. It is shown that, in contrast to the case of a sign step potential of Ref. [11], the spectrum consists of a finite set of bound-state solutions. An isolated solution from the Sturm–Liouville perspective is also present. With this methodology the whole relativistic spectrum is found, if the particle is massless or not. Nevertheless, bounded solutions do exist only under strict conditions. Interestingly, all of those bound-state solutions tend to disappear as the conditions for "spin and pseudospin symmetries" are approached. We also consider the limit where the smooth step potential becomes the sign step potential.

2. Scalar and vector potentials in the Dirac equation

The Dirac equation for a fermion of rest mass m reads

$$\left(\gamma^{\mu} p_{\mu} - Imc - V/c\right) \Psi = 0 \tag{1}$$

where $p_{\mu} = i\hbar \partial_{\mu}$ is the momentum operator, c is the velocity of light, I is the unit matrix, and the square matrices γ^{μ} satisfy the algebra $\{\gamma^{\mu}, \gamma^{\nu}\} = 2Ig^{\mu\nu}$. In $1+1$ dimensions Ψ is a 2×1 matrix and the metric tensor is $g^{\mu\nu} = \mathrm{diag}\,(1, -1)$. For vector and scalar interactions the matrix potential is written as

$$V = \gamma^{\mu} A_{\mu} + IV_{s}. \tag{2}$$

We say that A_{μ} and V_s are the vector and scalar potentials, respectively, because the bilinear forms $\bar{\Psi}\gamma^{\mu}\Psi$ and $\bar{\Psi}I\Psi$ behave like vector and scalar quantities under a Lorentz transformation, respectively. Eq. (1) can be written in the form

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \tag{3}$$