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Black hole temperature: Minimal coupling vs conformal coupling

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ABSTRACT

In this article, we discuss the propagation of scalar fields in conformally transformed spacetimes with either minimal or conformal coupling. The conformally coupled equation of motion is transformed into a one-dimensional Schrödinger-like equation with an invariant potential under conformal transformation. In a second stage, we argue that calculations based on conformal coupling yield the same Hawking temperature as those based on minimal coupling. Finally, it is conjectured that the quasi normal modes of black holes are invariant under conformal transformation.

1. Introduction

Having incorporated the quantum behavior of a propagating field in a fixed curved background and calculated the Bogoliubov coefficients, Hawking [1] discovered that black holes were not really black as they had been believed to be, but that they radiate with a perfect black body spectrum. Apart from Hawking's original calculation, there have been many derivations of the Hawking effect. Euclidean signature methods have been employed to show that the Hawking temperature is related to the periodicity in imaginary time [2]. Some authors have exploited the structure of trace anomaly to derive the Hawking radiation [3]. Another popular version is the tunneling mechanism that obtains

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the Hawking flux from a path integral across the horizon [4]. An alternative approach is to compute the reflection and absorption coefficients of quantum fields in the black hole spacetime and employing the connection between them to obtain the Bogoliubov coefficients which will ultimately yield the Hawking temperature [5–8],

$$\frac{R}{T} = \frac{1}{e^{\frac{\omega}{T_H}} - 1}. \tag{1}$$

Stephen Hawking showed that the area of event horizon never decreases in any classical physical processes [9]. The analogy between this and the second law of thermodynamics was inspiring for Bekenstein to associate entropy to the black holes, which is proportional to the area of the event horizon [10]. This is contrary to our intuitive expectation that the number of states is proportional to the volume of the system. Motivated by this, the holographic principle was proposed by Gerard t'Hooft [11] and it was later developed by Leonard Susskind [12]. It states that the information content in a macroscopic region of space can be represented by a theory that lives on the surface that encloses this region. The concrete examples of the holographic principle in the area of gravity are the black holes entropy and the AdS/CFT correspondence [13]. The AdS/CFT correspondence is the duality between the gravity living in five dimensional AdS spacetime and the super Yang–Mills conformal field theory happening in its boundary.

The study of Hawking temperature behavior under conformal transformation is a useful method for gaining insight into the conformal invariance of such gravitational systems as black holes. No convincing evidence is available to indicate that the conformal transformation is a symmetry of nature; however, it has been recently proposed that the conformal theory might be able to yield the evolution of the galaxies' dark matter [14]. Moreover, a number of authors [15] have argued that the outcomes of classical physical experiments should not be altered under such a transformation. This is contrary to the fact that this transformation does change the geometry. The Christoffel symbols and the curvature tensor are not invariant under this transformation. Besides, time like geodesics in one conformal frame is not necessarily geodesics in other conformal frames, but null rays remain geodesics although not necessarily affinely parameterized [16,17]. However, since the Einstein field equation is not conformally invariant, a conformally transformed black hole solution will not equally hold for the field equations of general relativity. Nonetheless, the event horizon remains unchanged under rescaling because it is a null hypersurface and the transformed black hole may still serve as a background on which Hawking radiation emerges and, thus, a temperature could be attributed to its event horizon. The equivalence of various thermodynamics quantities under conformal transformation has been explored in [18].

In this paper, we investigate the evolution of scalar fields via both conformally and minimally coupled equations of motion. Like the minimal coupling equation, the conformally coupled one can be transformed into a one-dimensional Schrödinger-like equation with a conformally invariant potential if suitable variable changes are considered. Jacobson and Kang have argued that Hawking temperature of the asymptotically flat and static black holes remains unchanged under conformal transformations that are identity at the infinity [19]. Here, we provide evidence in favor of the argument that the conformally coupled equation of motion gives the same Hawking temperature as does its minimally coupled counterpart and that the Hawking temperature is, therefore, invariant under the conformal transformation. We further discuss the method to identify the conformal factors that relate black holes to spacetimes with zero scalar curvature, and examine their behavior at asymptotic infinity. Furthermore, the conformal factors are obtained in some spacetimes. The minimal coupling choice is utilized to work out the Hawking temperature for some conformally deformed black holes that are not asymptotically flat with the conformal factors that are not identity at infinity to see that the outcomes are the same as those of the original black holes. Thus, it seems that the restrictions introduced in [19] are the sufficient conditions. Finally, it is conjectured that quasi normal modes may be invariant under conformal transformation.

The plan of this paper is as follows. The second section investigates the scalar field theory in curved spacetimes with minimal and conformal coupling cases. In Section 3, it will be argued that both the conformal and minimal couplings yield the same temperature for black holes. In