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Generalized contexts and consistent histories in quantum mechanics

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ABSTRACT

We analyze a restriction of the theory of consistent histories by imposing that a valid description of a physical system must include quantum histories which satisfy the consistency conditions *for all states*. We prove that these conditions are equivalent to imposing the compatibility conditions of our formalism of generalized contexts. Moreover, we show that the theory of consistent histories with the consistency conditions for all states and the formalism of generalized context are equally useful representing expressions which involve properties at different times.

1. Introduction

In quantum mechanics, the properties of a physical system are represented by closed subspaces of the Hilbert space. The orthocomplemented lattice structure of the set of properties allows defining the conjunction, the disjunction and the negation of properties. The probabilities for the properties at a given value of time are given by the Born rule.

The standard formalism of quantum mechanics does not give a meaning to conjunctions or disjunctions of properties corresponding to different times. However, there are situations in which it is necessary to relate properties at different times. For example, in the measurement process it is necessary to establish a link between the pointer position after the measurement and the previous value of some observable of the measured system. In the double slit experiment it is necessary to argue

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about the impossibility to say through which slit passed the particle before it is detected, producing a spot in a photographic plate.

In order to deal with conjunctions or disjunctions of properties at different times, R. Griffiths [1], R. Omnès [2], M. Gell-Mann and J. Hartle [3] have developed the theory of consistent histories. In this theory the allowed sets of quantum histories included in a description of the system must satisfy some consistency conditions. As the consistency conditions depend on the state of the system, the properties of the system which can be included in a valid description of the system also depend on the state. This is an odd situation compared with the standard formalism of quantum mechanics, where the allowed contexts of properties are all the possible distributive sublattices of the Hilbert space, which do not depend on the state.

Moreover, in the axiomatic approaches of the standard formalism of quantum mechanics, once the possible properties are identified, the states can be defined as functionals acting on the space of the properties, appearing after these observables in a somehow subordinated position [4,5].

We presented in previous papers the generalized context formalism [6,7]. In this formalism, well defined probabilities can be obtained for the conjunction of properties at different times, provided they satisfy some compatibility conditions. Compatible properties at different times are represented by commuting projectors when they are translated to a common time. This formalism was applied to the double slit experiment [7], to the logic of quantum measurements [8] and to quantum decay processes [9].

In this paper we explore the results that can be obtained from the theory of consistent histories by imposing the consistency conditions on all the states of the system. We obtain that the consistency conditions over all the states are equivalent to the compatibility conditions of our generalized contexts formalism. We also show that the theory of state-independent consistent histories and the theory of generalized context are equally useful representing expressions which involve properties at different times. Both formalism can represent the same expressions and the corresponding probabilities have the same value.

In Section 2 we present a brief description of the lattice of properties in standard quantum mechanics, emphasizing the notion of contexts of properties for a fixed value of time. The main features of the theory of consistent histories are given in Section 3, pointing out the motivations for considering the consistency conditions for all states. In Section 4 we relate the consistency conditions for all states with the commutation of the projectors corresponding to the time translation of properties to a common time. In Section 5 we summarize our previously developed formalism of generalized contexts. The consistency conditions for all states and the formalism of generalized contexts are compared in Section 6. The main conclusions are given in Section 7.

2. Quantum contexts

In quantum mechanics, each isolated physical system is associated with a Hilbert space $\mathcal{H}$ and a Hamiltonian operator $H : \mathcal{H} \longrightarrow \mathcal{H}$. The *state* of the system is represented by a nonnegative, normalized and self adjoint *density operator* $\rho : \mathcal{H} \longrightarrow \mathcal{H}$.

The time evolution of the state is generated by the Liouville–von Neumann equation. If ρ_t is the density operator representing the state at time t, the state at a different time t' is represented by

$$\rho_{t'} = U(t', t)\ \rho_t\ U(t', t)^{-1}, \tag{1}$$

where $U(t', t) = e^{-\frac{i}{\hbar}H(t'-t)}$ is the unitary time translation operator.

The *properties* of a quantum system are represented by closed vector subspaces of the Hilbert space $\mathcal{H}$. As for each closed subspace V there exists only one orthogonal projection operator $\Pi_V : \mathcal{H} \longrightarrow \mathcal{H}$ such that $V = \Pi_V\ \mathcal{H}$, each property V can also be represented by the projector Π_V.

The set of all closed vector subspaces of $\mathcal{H}$, with the *partial order relation* given by the set inclusion ($\subset$), is an orthocomplemented nondistributive lattice. The *supremum* of V and V' is given by $\text{Sup}(V, V') = V + V'$ and the *infimum* is given by $\text{Inf}(V, V') = V \cap V'$. The *universal property* is represented with the whole space $\mathcal{H}$ and the *zero property* is represented with the subspace $\{0_{\mathcal{H}}\}$, where $0_{\mathcal{H}}$ is the zero element of $\mathcal{H}$. The *complement* of a property V is the orthogonal complement $V^{\perp}$ of the subspace V in $\mathcal{H}$.