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A Kaluza–Klein description of geometric phases in graphene

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ABSTRACT

In this paper, we use the Kaluza–Klein approach to describe topological defects in a graphene layer. Using this approach, we propose a geometric model allowing us to discuss the quantum flux in K-spin subspace. Within this model, the graphene layer with a topological defect is described using a four-dimensional metric, where the deformation produced by the topological defect is introduced via the three-dimensional part of a metric tensor, while an Abelian gauge field is introduced via an extra dimension. We use this new geometric model to discuss the arising of topological quantum phases in a graphene layer with a topological defect.

1. Introduction

The concept of extra dimensions, which is treated now as one of the most popular concepts in high energy physics, has a long history. Initially [1,2], the hypothesis of extra dimensions was introduced as an attempt to unify gravity and electromagnetism. Despite this attempt not succeeding and having been abandoned, the idea of introducing additional spacetime dimensions as a possible instrument allowing one to construct unified physical theories has found wide applications in quantum field theory, especially due to the development of string theory, which is well-known to be consistent in a space with extra dimensions [3].

The essence of the Kaluza–Klein approach in its modern form is as follows. Let us suggest that spacetime involves, besides the usual dimensions, compact extra dimensions. As a consequence, all fields defined in this space can be expanded in Fourier series with respect to the extra coordinates. Different Fourier modes arising throughout this expansion possess different masses depending on the sizes of the compact extra dimensions; thus, in principle, an infinite tower of new particles with

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an infinite spectrum of masses can arise. Besides the string applications, the Kaluza–Klein approach turned out to provide an efficient description of black holes (for a review on Kaluza–Klein theory, see [4]).

Graphene, whose different aspects have been studied earlier in a number of papers [5–9], is a physical system whose description essentially involves topological defects. Different aspects of graphene have been studied with the use of the gauge fields approach [7–9], finite temperature methods [10], Berry phases arising from the presence of dislocations in the graphene layer [11], and the parallel transport of Dirac fermions in the presence of torsion and curvature [12]. Recently, several investigations have established the physical similarity between gravity and some models used in condensed matter theory [13–15]. It is well-known that smooth deformations of graphene sheets produce a gauge field similar to the electromagnetic one [16,17], and topological defects in graphene can be interpreted as a source of a non-Abelian gauge field [18–21]. On the basis of this bridge between the physics of graphene and properties of the gauge and gravitational fields, in this work we use the Kaluza–Klein theory to describe a graphene layer with a topological defect. The success of applying the quantum field theory concepts in condensed matter theory leads us to hope that using the Kaluza–Klein theory for describing certain condensed matter models could be very efficient. In particular, one very promising idea consists in applying the Kaluza–Klein approach to graphene, where the extra dimension concept naturally emerges in the case of the presence of a topological defect.

In this paper, we extend the two-dimensional metric which describes a topological defect in a graphene layer to a three-dimensional metric by adding an extra compact dimension to describe the Fermi-point degree of freedom, and demonstrate that we can calculate the geometric phase acquired by the wavefunction of a massless fermion which arises from the mixing of the Fermi points, and the parallel transport of a spinor around the apex of a defect in a graphene layer.

"Geometric quantum phases" is a term introduced by Berry [22] to describe the phase shifts acquired by the wavefunction of a quantum particle in adiabatic evolution. A well-known quantum phase that belongs to this more general class of phases is the Aharonov–Bohm effect [23]. Within this effect, the wavefunction of the electron acquires a topological quantum phase circulating in a solenoid. The electron moves in a region where the magnetic field is zero, and still feels the influence of the magnetic field via the vector potential in the phase acquired by the wavefunction describing the motion of the electron. Aharonov and Anandan [24] extended the study of geometric quantum phases to any cyclic evolution, and the phase shift associated with any cyclic evolution is known as the Aharonov–Anandan quantum phase. The study of geometric phases in a quantum system has attracted a great deal of attention in recent years [25,26]; the most important quantum effect is the Aharonov–Bohm effect [23]. In graphene, the geometric phase arising from the presence of a disclination in a graphene layer has been pointed out in [20,21]. In this paper, we extend the two-dimensional metric which describes a topological defect in a graphene layer to a three-dimensional metric by adding an extra compact dimension to describe the Fermi-point degree of freedom, and demonstrate that we can calculate the geometric phase acquired by the wavefunction of a massless fermion which arises from the mixing of the Fermi points and the parallel transport of a spinor around the apex of a defect in a graphene layer.

This paper is organized as follows. In Section 2, we a give a brief review of graphene, and the appearance of quantum fluxes due to the presence of a topological defect. In Section 3, we introduce the concept of an extra dimension from the Kaluza–Klein theory [1,2] for graphene, and discuss how to obtain the quantum flux from the K-spin part of graphene from a geometrical point of view. Finally, in Section 4, we present our conclusions.

2. Graphene: a brief review

In this section, we give a brief review of graphene and the arising of quantum phases caused by the presence of a topological defect called a disclination [13–15,27,28]. The conduction band of graphene, which consists of a graphite layer, can be described by using the tight-binding model [18–21]. A graphene layer is a two-dimensional material formed by an isolated layer of carbon atoms arranged in a honeycomb lattice. We describe this structure by means of two sublattices A and B, where the unit cell and the vector of the unit cell are represented in Fig. 1.