Annals of Physics 327 (2012) 2972–3047

Contents lists available at SciVerse ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Non-abelian symmetries in tensor networks: A quantum symmetry space approach

Andreas Weichselbaum

Department of Physics, Arnold Sommerfeld Center for Theoretical Physics, and Center for NanoScience, Ludwig-Maximilians-Universität, 80333 Munich, Germany

ARTICLE INFO

Article history:
Received 10 March 2012
Accepted 8 July 2012
Available online 24 July 2012

Keywords:
Non-abelian symmetries
Clebsch–Gordan coefficients
Lie algebra
Numerical renormalization group
Density matrix renormalization group
Tensor networks

ABSTRACT

A general framework for non-abelian symmetries is presented for matrix-product and tensor-network states in the presence of well-defined orthonormal local as well as effective basis sets. The two crucial ingredients, the Clebsch–Gordan algebra for multiplet spaces as well as the Wigner–Eckart theorem for operators, are accounted for in a natural, well-organized, and computationally straightforward way. The unifying tensor-representation for quantum symmetry spaces, dubbed QSpace, is particularly suitable to deal with standard renormalization group algorithms such as the numerical renormalization group (NRG), the density matrix renormalization group (DMRG), or also more general tensor networks such as the multi-scale entanglement renormalization ansatz (MERA). In this paper, the focus is on the application of the non-abelian framework within the NRG. A detailed analysis is presented for a fully screened spin-3/2 three-channel Anderson impurity model in the presence of conservation of total spin, particle–hole symmetry, and SU(3) channel symmetry. The same system is analyzed using several alternative symmetry scenarios based on combinations of $U(1)_{\text{charge}}$, $SU(2)_{\text{spin}}$, $SU(2)_{\text{charge}}$, $SU(3)_{\text{channel}}$, as well as the enveloping symplectic Sp(6) symmetry. These are compared in detail, including their respective dramatic gain in numerical efficiency. In the Appendix, finally, an extensive introduction to non-abelian symmetries is given for practical applications, together with simple self-contained numerical procedures to obtain Clebsch–Gordan coefficients and irreducible operators sets. The resulting QSpace tensors can deal with any set of abelian symmetries together with arbitrary non-abelian symmetries with compact, *i.e.* finite-dimensional, semi-simple Lie algebras.

E-mail address: Andreas.Weichselbaum@physik.lmu.de.

0003-4916/$ – see front matter © 2012 Elsevier Inc. All rights reserved.
doi:10.1016/j.aop.2012.07.009

Contents

1. Introduction

Numerical methods for strongly correlated quantum-many-body systems are confronted with exponentially large Hilbert spaces. With a limited number of exact analytical solutions at hand and with perturbative treatments for low-energy or ground-state physics often insufficient, a certain systematic treatment with respect the Hilbert space is required. Besides quantum Monte Carlo approaches, that explore quantum systems in a stochastic way [1], a systematic state space decimation is provided by renormalization group (RG) techniques such as the density matrix renormalization group (DMRG) [2] or the numerical renormalization group (NRG) [3], both highly efficient for quasi-one-dimensional systems, and since non-perturbative, considered essentially exact.

Quantum-many-body Hilbert spaces are built from the direct product of the state spaces of the participating individual particles. As such particle statistics plays an essential role. While the