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Entropy excess in strongly correlated Fermi systems near a quantum critical point

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ABSTRACT

A system of interacting, identical fermions described by standard Landau Fermi-liquid (FL) theory can experience a rearrangement of its Fermi surface if the correlations grow sufficiently strong, as occurs at a quantum critical point where the effective mass diverges. As yet, this phenomenon defies full understanding, but salient aspects of the non-Fermi-liquid (NFL) behavior observed beyond the quantum critical point are still accessible within the general framework of the Landau quasiparticle picture. Self-consistent solutions of the coupled Landau equations for the quasiparticle momentum distribution $n(p)$ and quasiparticle energy spectrum $\epsilon(p)$ are shown to exist in two distinct classes, depending on coupling strength and on whether the quasiparticle interaction is regular or singular at zero momentum transfer. One class of solutions maintains the idempotency condition $n^2(p) = n(p)$ of standard FL theory at zero temperature T while adding pockets to the Fermi surface. The other solutions are characterized by a swelling of the Fermi surface and a flattening of the spectrum $\epsilon(p)$ over a range of momenta in which the quasiparticle occupancies lie between 0 and 1 even at $T = 0$. The latter, non-idempotent solution is revealed by analysis of a Poincaré mapping associated with the fundamental Landau equation connecting $n(p)$ and $\epsilon(p)$ and validated by solution of a variational condition that yields the symmetry-preserving ground state. Significantly, this extraordinary solution carries the burden of a large temperature-dependent excess entropy down to very low temperatures, threatening violation of the Nernst Theorem. It

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is argued that certain low-temperature phase transitions, notably those involving Cooper-pair formation, offer effective mechanisms for shedding the entropy excess. Available measurements in heavy-fermion compounds provide concrete support for such a scenario.

1. Introduction

Landau Fermi liquid (FL) theory [1–4] is universally recognized as a cornerstone of modern low-temperature condensed matter physics. This theory predicts that the magnetic susceptibility $\chi(T)$ becomes independent of temperature T as $T \to 0$. It also predicts that in this regime the entropy $S(T)$ varies linearly with T, implying the same behavior for the specific heat $C(T) = TdS(T)/dT$ and thermal expansion coefficient $\alpha = l^{-1}\partial l/\partial T \propto -\partial S(T)/\partial P$. These predictions are in excellent agreement with available experimental data on conventional Fermi liquids, notably three-dimensional (3D) liquid ^{3}He and the electron liquid present in ordinary metals.

However, beginning the mid-1990s experimental studies of many-fermion systems have yielded abundant evidence of the failure of the FL picture upon entry into the regime of strong correlations among the particles. Such non-Fermi-liquid (NFL) behavior was first revealed and explored in experiments [5–9] on films of liquid ^{3}He and found subsequently in electron systems in solids, especially heavy-fermion compounds [10–12]. One of the striking features of the observed NFL behavior is that as the temperature T drops to zero, both the spin susceptibility $\chi(T)$ and the Sommerfeld ratio $\gamma(T) = C(T)/T \equiv dS(T)/dT$ diverge when the density (or doping) reaches a critical value, while the Sommerfeld–Wilson ratio $R_{SW} = \chi(T)/\gamma(T)$ changes much more slowly.

The corresponding point in the Lifshitz phase diagram showing the sequence of different phases that replace one another at $T = 0$ under variation of a suitable control parameter (coupling strength, density, doping) is generally called the quantum critical point (QCP). If the QCP were the end point, from finite T values, of a line of transition points $T_c(B)$ associated with some control parameter B, then according to the theory of second-order phase transitions, divergence of $\gamma(T)$ and $\chi(T)$ at $T \to 0$ would be specified by different critical indexes. For example, in the Landau theory of second-order phase transitions, the critical index ς characterizing the divergence of the Sommerfeld ratio is zero, while that for the divergence of the magnetic susceptibility χ is 4/3. In Wilson theory, the critical indexes have almost the same values as in Landau theory [13]. However, experiment deviates strongly from such theoretical predictions. For example, the critical-index value $\varsigma = 0.38$ has been determined [14] for the compound $YbRh_2Si_2$, which belongs to a family of heavy-fermion metals comprehensively studied by Steglich and his collaborators [15–19].

Further, the posited second-order phase transition, attributed as usual to critical spin fluctuations, possesses unusual properties: the magnetic moment specifying the ordered state of the metal $YbRh_2Si_2$ is extremely small, $\simeq 10^{-3}\ \mu_B$, as if the order parameter were hidden [20]. Furthermore, in dense ^{3}He films, where the emergent NFL behavior was documented for the first time, experiment has not identified any related second-order phase transition. Thus the theory of second-order phase transitions is hardly relevant to explanation of the observed NFL behavior.

Consider, on the other hand, that within FL theory the quantities χ and γ are both proportional to the density of states $N(0)$. In turn, $N(0)$ is a linear function of the effective mass M^*, which specifies, via the dispersion $d\epsilon(p \to p_F)/dp = p_F/M^*$, the spectrum $\epsilon(p)$ of single-particle excitations near the Fermi surface. Thus, the NFL behavior emerging at the QCP can be explained within the broader framework of FL theory by attributing the common divergence of χ and γ to divergence of the effective mass M^* at the QCP.

The NFL behavior of the entropy S, which is the subject of the present article, is exhibited primarily in the divergence, at the QCP, of the thermal expansion coefficient $\alpha(T \to 0)$. The most challenging data have been obtained for a group of heavy-fermion metals [18]. For $CeCoIn_5$ in particular, $\alpha(T \to 0)$ is found to be almost independent of temperature and exceeds typical values for ordinary metals by a huge factor 10^3–10^4, implying a corresponding enhancement of the entropy itself (see Fig. 6).