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On the nonperturbative solution of Pauli–Villars-regulated light-front QED: A comparison of the sector-dependent and standard parameterizations

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ABSTRACT

We consider quantum electrodynamics quantized on the light front in Feynman gauge and regulated in the ultraviolet by the inclusion of massive, negative-metric Pauli–Villars (PV) particles in the Lagrangian. The eigenstate of the electron is approximated by a Fock-state expansion truncated to include one photon. The Fock-state wave functions are computed from the fundamental Hamiltonian eigenvalue problem and used to calculate the anomalous magnetic moment, as a point of comparison. Two approaches are considered: a sector-dependent parameterization, where the bare parameters of the Lagrangian are allowed to depend on the Fock sectors between which the particular Hamiltonian term acts, and the standard choice, where the bare parameters are the same for all sectors. Both methods are shown to require some care with respect to ultraviolet divergences; neither method can allow all PV masses to be taken to infinity. In addition, the sector-dependent approach suffers from an infrared divergence that requires a nonzero photon mass; due to complications associated with this divergence, the standard parameterization is to be preferred. We also show that the self-energy effects obtained from a two-photon truncation are enough to bring the standard-parameterization result for the anomalous moment into agreement with experiment within numerical errors. This continues the development of a method for the nonperturbative solution of strongly coupled theories, in particular quantum chromodynamics.

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1. Introduction

The nonperturbative solution of quantum field theories has proven to be a difficult task. The method that has had the most success to date, lattice gauge theory [1], has attained this success only after a long period of development, with a great number of technical innovations along the way. What is more, the level of success that can be achieved is inherently limited by the lack of direct contact with wave functions for bound-state constituents and by the Euclidean formulation. The limitation of Euclidean formulation is shared by the method of Dyson–Schwinger equations [2], which are coupled equations for the n-point Euclidean Green's functions where bound states appear as poles in the propagators. Solution of the infinite system requires truncation and a model for the highest n-point functions.

In order to calculate wave functions directly in a Minkowskian formulation, there has been an effort for a number of years to develop a Hamiltonian approach in light-cone quantization [3]. Although the ultimate objective is to be able to solve for the bound states of QCD, most of the development has been in QED and simpler theories, such as Yukawa theory and ϕ^4 theory. Many two-dimensional theories have been solved; however, success with four-dimensional theories has been limited by the need for a consistent regularization and renormalization scheme and by the large size of the numerical calculation. There is also a related light-front lattice Hamiltonian formulation, known as the transverse lattice method [4], which we do not consider here.

A regularization scheme that has been useful in doing perturbative light-cone calculations is the alternate denominator method of Brodsky et al. [5]. Because of its success, one would naturally consider extending this approach to nonperturbative calculations. Unfortunately, application of the method requires explicit identification of light-cone energy denominators, which are only implicit in the coupled equations of the nonperturbative mass eigenvalue problem. One could instead use the alternate denominator method to construct counterterms, and then incorporate the counterterms in the Hamiltonian for the nonperturbative calculation. However, these counterterms would be limited to a particular order in the coupling while the nonperturbative problem sums a partial set of contributions to all orders. Instead, one needs a method that generates counterterms to all orders, but no more than what is needed.

A method for regularization that has proven quite useful is Pauli–Villars (PV) regularization [6], which was developed and tested in a series of calculations [7–14]. The key idea is to include enough PV fields in the Lagrangian to regulate the theory perturbatively and, where possible, maintain symmetries.[1] The derived light-front Hamiltonian then defines the nonperturbative bound-state problem. As for the eigenstate, it is approximated by a truncated Fock-state expansion. Then the mass eigenvalue problem leads to coupled integral equations for the Fock-state wave functions. The PV particles appear in the Fock states and, through chosen negative metrics, bring about the subtractions necessary to regulate the integral equations. A possible formulation for QCD along these lines has been given by Paston et al. [16].

Two methods of parameterization are in use. One is the standard choice, where the bare parameters are those of the regulated Lagrangian. The other is a sector-dependent parameterization [17], where the bare parameters are allowed to depend on the Fock sector(s) on which the Hamiltonian acts. In either case, the parameters are fixed by constraints from observables and from symmetry restorations. The standard scheme has been used extensively in studies of PV regularization [7–14]. The sector-dependent scheme was first systematically applied to QED by Hiller and Brodsky [18], though they did not consider a sector-dependent vertex mass. More recently, it was investigated by Karmanov et al. [19]. In order to better understand how to proceed with the large-scale numerical calculations that need to be done, a comparison of these approaches needs to be made.

Both parameterizations require some care, particularly with respect to uncanceled divergences. In the standard method, an uncanceled divergence appears in the following way [12]. The results for a generic physical quantity, such as the electron's anomalous moment, will be of the form

[1] The introduction of PV partners to the fields of a theory has recently been used to define extensions of the Standard Model that offer a solution to the hierarchy problem [15].