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Three-dimensional Casimir piston for massive scalar fields

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ABSTRACT

We consider Casimir force acting on a three-dimensional rectangular piston due to a massive scalar field subject to periodic, Dirichlet and Neumann boundary conditions. Exponential cut-off method is used to derive the Casimir energy. It is shown that the divergent terms do not contribute to the Casimir force acting on the piston, thus render a finite well-defined Casimir force acting on the piston. Explicit expressions for the total Casimir force acting on the piston is derived, which show that the Casimir force is always attractive for all the different boundary conditions considered. As a function of a – the distance from the piston to the opposite wall, it is found that the magnitude of the Casimir force behaves like $1/a^4$ when $a \to 0^+$ and decays exponentially when $a \to \infty$. Moreover, the magnitude of the Casimir force is always a decreasing function of a. On the other hand, passing from massless to massive, we find that the effect of the mass is insignificant when a is small, but the magnitude of the force is decreased for large a in the massive case.

1. Introduction

Casimir effect associated with piston geometry has attracted considerable interest since its introduction by Cavalcanti [1] a few years ago. The main attraction of Casimir piston is that such a geometric setup can resolve a serious divergent problem that plagues the Casimir calculations. In the conventional calculations of Casimir force inside a confined region such as a rectangular box, the nontrivial contribution of vacuum energy outside the box has been ignored, and the surface divergent terms which depend on the dimensions and geometry are discarded to obtain finite results under the pretext of some regularization schemes. It has been shown that such surface divergence cannot be removed by renormalization of physical parameters of the theory [2,3]. Calvacanti [1] considered

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a new geometric configuration – a two-dimensional rectangular piston, and showed that for a massless scalar field subject to Dirichlet boundary conditions, the divergent part of the Casimir force acting on the piston due to the two regions separated by the piston cancel with each other and the net result is a finite attractive Casimir force without the surface divergence. This result has stimulated an interest in studying the Casimir force on pistons with different geometric setups and boundary conditions. In particular, Hertzberg et al. [4] have studied the Casimir effect for electromagnetic fields in three-dimensional rectangular pistons with perfectly conducting walls. It was found that the Casimir force is always attractive. This work was generalized in [5] where finite temperature effect was taken into account and pistons with general cross sections were considered. Around the same time, Marachevsky also carried out a similar investigation using a different approach [6–8]. In [9], Edery studied the case of a massless scalar field with Dirichlet boundary conditions for three-dimensional rectangular pistons. He also found that the force on the piston is always attractive, in contrast to the fact that the regularized Casimir force for a three-dimensional rectangular cavity for massless scalar field with Dirichlet boundary conditions can be attractive or repulsive depending on the relative size of the cavity. This work was later generalized in [10] to massless scalar fields with Dirichlet and Neumann boundary conditions in any dimensions. In all the above scenarios, the Casimir force was found to be attractive. In [11], Barton found that when weakly reflecting dielectric materials are used, the Casimir force on the three-dimensional piston can become repulsive when the plate separation is sufficiently large. On the other hand, the work of Zhai and Li [12] showed that in the case of mixed boundary conditions (one plate with Dirichlet boundary conditions and one with Neumann boundary conditions), the Casimir force on a rectangular piston in one, two and three dimensions is always repulsive. Recently, Casimir effect was investigated for electromagnetic fields with perfect magnetic conductor conditions in rectangular pistons of arbitrary dimensions [13] and for massless scalar fields with Dirichlet boundary conditions in pistons inside space-time with extra compactified dimensions [14]. In [15], the piston setup was generalized to a setup called pistol as a closer analog of a rectangular cavity.

In this work, we consider massive scalar field in a three-dimensional rectangular piston subject to periodic boundary conditions, Dirichlet boundary conditions as well as Neumann boundary conditions. We use exponential cut-off method to compute the cut-off dependent Casimir energy of the piston system. We show that the Casimir energy has a divergent part that does not depend on the position of the piston, and therefore, the force acting on the piston is finite, without any regularization. Explicit formulas for the Casimir force are derived, which can be written as an infinite convergent sums of Bessel functions. From the formulas, it is easy to deduce that the Casimir force is always attractive under all the different boundary conditions. We also proved a stronger result: as a function of a – the separation between the piston and the opposite wall, the magnitude of the Casimir force behaves like $1/a^4$ when $a \to 0^+$ and it decays exponentially when $a \to \infty$. Moreover, the magnitude of the force is always decreasing from infinity to 0. The mass effect to the Casimir force is considered and it is found that the mass effect is more significant when a is large. Some numerical simulations have been carried out.

In this paper, we use the units with $\hbar = c = 1$.

2. Cancelation of divergence in three-dimensional Casimir piston

Consider a three-dimensional rectangular piston in the form of a rectangular cavity $[0, L_1] \times [0, L_2] \times [0, L_3]$ separated by a plane $x_1 = a$ (the piston) into two regions: left region $[0, a] \times [0, L_2] \times [0, L_3]$ and right region $[a, L_1] \times [0, L_2] \times [0, L_3]$ (see Fig. 1). At the end we are going to let $L_1 \to \infty$ so that the exterior region becomes an open region. We want to compute the Casimir force acting on the piston due to a massive ($m > 0$) scalar field $\phi(\mathbf{x}, t)$ satisfying the Klein–Gordon equation in Minkowski space-time:

$$\left(\Box + m^2\right)\phi(\mathbf{x}, t) = 0, \quad (\mathbf{x}, t) \in \mathbb{R}^{3+1}, \tag{1}$$

where

$$\Box = \frac{\partial^2}{\partial t^2} - \sum_{i=1}^{3} \frac{\partial^2}{\partial x_i^2}.$$