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Time reparameterization in Bianchi type I spinor cosmology

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Abstract

The problem of time reparameterization is addressed at both the classical and quantum levels in a Bianchi-I universe in which the matter source is a massive Dirac spinor field. We take the scale factors of the metric as the intrinsic time and their conjugate momenta as the extrinsic time. A scalar character of the spinor field is identified as a representation of the extrinsic time. The construction of the field equations and quantization of the model is achieved by solving the Hamiltonian constraint after time identification has been dealt with. This procedure leads to a true Hamiltonian whose exact solutions for the above choices of time are presented.

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Keywords: Time reparameterization; Bianchi type I; Quantum cosmology

1. Introduction

Standard cosmological models based on classical general relativity have no convincing precise answer for the presence of the so-called "Big-Bang" singularity. Any hope of dealing with such singularities would be in vein unless a reliable quantum theory of gravity can be constructed. In the absence of a full theory of quantum gravity, it would be useful to describe the quantum states of the universe within the context of quantum cosmology,

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introduced in the works of DeWitt [1] and later Misner [2]. In this formalism which is based on the canonical quantization procedure, one first freezes a large number of degrees of freedom and then quantizes the remaining ones. The quantum state of the universe is then described by a wave function in the mini-superspace, a function of the 3-geometry of the model and matter fields presented in the theory, satisfying the Wheeler–DeWitt (WD) equation. In more recent times such works have been the focus of an active area of research with different approaches, [3–11], see also [12] for a review. In references [13], canonical quantization is applied to many models with different matter fields as the sources of gravity.

As are well known, quantum cosmology suffers from a number of problems, namely the construction of the Hilbert space to define a positive definite inner product of the solutions of the WD equation, the operator ordering problem and also most importantly, the problem of time. The wave function in the WD equation is independent of time, i.e., the universe has a static picture in this scenario. This problem was first addressed in [1] by DeWitt himself. However, he argued that the problem of time should not be considered as a hindrance in the sense that the theory itself must include a suitable well-defined time in terms of its geometry or matter fields. In this scheme time is identified with one of the characters of the geometry, usually the scale factors of the geometry and is referred to as the intrinsic time, or with the momenta conjugate to the scale factors, or even with a scalar character of matter fields coupled to gravity in any specific model, known as the extrinsic time.

In general, the crucial problem in canonical quantum gravity is the presence of constraints in the gravitational field equations. Identification of time with one of the dynamical variables depends on the method we use to deal with theses constraints. Different approaches arising from these methods have been investigated in detail in [14]. The issue of time in canonical general relativity is also extensively discussed in [15]. As has been discussed in [14], time may be identified before or after quantization has been done. There are approaches, on the other hand, in which time has no fundamental role. For a more modern review of the problem of time and other related problems in quantum cosmology see [16]. The details of time identification procedure in terms of various dynamical variables of the theory before quantization is done has been investigated in [17] where a Robertson–Walker universe filled with a scalar field is quantized. Also in [18] a choice of time in terms of a massless scalar field is discussed in a Bianchi-I classical cosmology based on the method developed in [19].

One of the common candidates of time in the above works is the matter field present in the theory, that is, time is identified with a scalar character of matter. In the case of a scalar field as the source of gravity, the scalar field itself can play the role of time as is the case in [17,18]. Another matter field which has occasionally been studied in the literatures is the massless or massive spinor field as the source of gravity. In general, theories studying spinor fields coupled to gravity result in Einstein–Dirac systems which are not easy to solve. The quantized Robertson–Walker or Bianchi-I universe filled with a spinor field are studied in [20–25]. For a general discussion on the possibility that classical homogeneous spinor fields might play the role of matter in cosmology, the reader is referred to [26].

In this paper we deal with classical and quantum cosmology of a model in which a classical massive spinor field is coupled to gravity in a Bianchi type I space-time. What we mean by a classical spinor field is a set of four complex-valued space-time functions that transform according to the spinor representation of the Lorentz group. The existence of such fields is crucial in our work since in spite of fact that fermions are described by quan-