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Zero-temperature Kosterlitz–Thouless transition in a two-dimensional quantum system

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Abstract

We construct a local interacting quantum dimer model on the square lattice, whose zero-temperature phase diagram is characterized by a line of critical points separating two ordered phases of the valence bond crystal type. On one side, the line of critical points terminates in a quantum transition inherited from a Kosterlitz–Thouless transition in an associated classical model. We also discuss the effect of a longer-range dimer interaction that can be used to suppress the line of critical points by gradually shrinking it to a single point. Finally, we propose a way to generalize the quantum Hamiltonian to a dilute dimer model in presence of monomers and we qualitatively discuss the phase diagram.

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Keywords: Quantum dimer model; Quantum criticality; Kosterlitz–Thouless transition; Conformal field theory; Stochastic Matrix Form decomposition

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1. Introduction

Dimer models are of interest to a variety of scientific disciplines from chemistry to mathematics and physics. In chemistry, dimers are used, for example, to model molecules deposited on crystalline surfaces and to study their thermodynamic properties [1]. In mathematics, dimers are often used to construct combinatorial and folding problems such as the domino tiling of a two-dimensional ($2D$) plane [2]. In physics, dimer models have been elevated from problems in classical statistical physics [3–10], to problems in quantum statistical physics [11–39], with the advent of high-T_{c} superconductivity. In particular, quantum dimer models can provide examples of strongly correlated quantum systems for which the zero-temperature phase diagram is characterized by exotic quantum phase transitions that fall out of the classification of phase transitions proposed by Landau [40,41]. Furthermore, the finite-temperature phase diagram of quantum dimer models might give some insight into the phenomenological observation that scaling laws extend to surprisingly high temperatures in some strongly correlated systems [42,43].

In this paper, we show how dimer models can be used as a laboratory to construct quantum Hamiltonians displaying phase transitions that cannot be understood in terms of a local order parameter, i.e., phase transitions that cannot be encoded by an effective theory of the Landau–Ginzburg type, a topic of renewed interest in condensed matter physics [40,41]. Perhaps the most famous counter example to a phase transition described with a Landau–Ginzburg action for a local order parameter is the Kosterlitz–Thouless (KT) transition. The KT transition is a weak essential singularity of the free energy for a phase-like order parameter with support in $2D$ Euclidean space. It is interpreted as the unbinding of topological defects (vortices) in the order parameter. The main result of this paper is the construction of a quantum dimer model with *local interactions* on the square lattice Eqs. (13) and (14) that undergoes a quantum phase transition of the KT type when measured by the spatial decay of equal-time correlation functions.

It is well known that quantum phase transitions can be of the KT type in $1D$ systems with dynamical exponent $z = 1$ relating the scaling in space to the scaling in time. For example, a $1D$ Luttinger liquid can be unstable to a charge-ordered density wave through a KT transition. This is so because the quantum field theory describing the quantum phase transition for interacting fermions is related through bosonization to a scalar field theory, the Sine–Gordon model. Analytical continuation of time to imaginary time can be used to turn (Minkowski) space-time into $2D$ Euclidean space while the Poincaré symmetry group becomes symmetry under translations and rotations. Evidently, if a quantum phase transition can be described by a local quantum field theory in $D + 1$ space-time that turns into a local classical action undergoing a classical phase transition in $D + 1$ Euclidean space upon analytical continuation of time to imaginary time [44], then this quantum phase transition cannot be associated to a KT transition when $z = 1$ and $D \geqslant 2$. Viewed against this no-go theorem, it is remarkable that some equal-time correlation functions of a $2D$ quantum dimer model with local interactions and defined on the square lattice Eqs. (13) and (14) share the hallmarks of a KT transition.

Since the quantum Hamiltonian of our $2D$ lattice model, defined in Eqs. (13) and (14), has only local interactions, it is safe to argue that unequal-time correlations should show algebraic behavior if the equal-time correlations do so. The reason is that a local Hamiltonian with algebraic spatial correlations should be gapless. A rigorous proof of this statement was given by Hastings in Ref. [45]. The converse statement is known not to be true,