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Line of magnetic monopoles and an extension of the Aharonov–Bohm effect

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ABSTRACT

In the Landau problem on the two-dimensional plane, physical displacement of a charged particle (i.e., magnetic translation) can be induced by an in-plane electric field. The geometric phase accompanying such magnetic translation around a closed path differs from the topological phase of Aharonov and Bohm in two essential aspects: The particle is in direct contact with the magnetic field and the geometric phase has an opposite sign from the Aharonov–Bohm phase. We show that magnetic translation on the two-dimensional cylinder implemented by the Schrödinger time evolution truly leads to the Aharonov–Bohm effect. The magnetic field normal to the cylinder's surface corresponds to a line of magnetic monopoles of uniform density whose simulation is currently under investigation in cold atom physics. In order to characterize the quantum problem, one needs to specify the value of the magnetic flux (modulo the flux unit) that threads but not in touch with the cylinder. A general closed path on the cylinder may enclose both the Aharonov–Bohm flux and the local magnetic field that is in direct contact with the charged particle. This suggests an extension of the Aharonov–Bohm experiment that naturally takes into account both the geometric phase due to local interaction with the magnetic field and the topological phase of Aharonov and Bohm.

1. Introduction

The Dirac monopole [1] and the Aharonov–Bohm effect [2] are fundamental subjects in quantum physics. They demonstrate how topological conditions may dominate the behavior of quantum

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systems that interact with the electromagnetic field. They also have intimate connections with the Berry phase (or geometric phase) [3] which has wide applications in many different areas [4] in physics. While the Aharonov–Bohm effect has been verified experimentally, synthetic magnetic fields such as the field of a Dirac monopole or a line of monopoles are explored only recently in experiments involving Bose–Einstein condensates [5,6]. This has provided impetus for theoretical investigations of new possibilities provided by such experimental progress.

A unique situation where a line of magnetic monopoles plays an essential role is a charged particle with charge q on a two-dimensional cylinder subjected to a magnetic field B normal to the cylinder's surface, which we refer to as the Landau problem on the cylinder [7]. The magnetic field can be seen as produced by a line of monopoles of uniform density located at the central axis of the cylinder. In order to completely describe the quantum problem, one needs to specify the vector potential in the Hamiltonian. For the same B, the vector potential can be classified into gauge equivalence classes according to the magnetic flux ϕ that threads the cylinder modulo hc/q.[1] Different classes of the vector potential represent different physical situations that are experimentally distinguishable. So one needs both B and ϕ modulo hc/q to completely characterize the Landau problem on the cylinder.

Laughlin applied such a model in his treatment of the integer quantum Hall effect [9]. Connected with Laughlin's analysis is the fact that the Landau problem on the cylinder has reduced quantum mechanical symmetry so that translation symmetry in the longitudinal direction is discrete. Laughlin's derivation of the integer quantum Hall effect and the reduction of symmetry are both related to the role played by hc/q, a quantity that can be reminiscent of the Aharonov–Bohm effect. However, unlike the Aharonov–Bohm effect, they have no bearing on the value ϕ modulo hc/q. This is ultimately due to the fact that the quantum Hall conductivity and the step size of translation symmetry in the longitudinal direction are changed if hc/q has a different value but neither is affected if one starts out with a different ϕ. This point will be discussed in detail in Section 4.

Our present work has its origin in the Landau problem on the two-dimensional plane. Imagine a quantum state, say a ground state Ψ_0, that is a Gaussian in its probability distribution. Now consider magnetic translation of the wave around a loop on the plane. (The loop can be taken as the trajectory of the center of Ψ_0 where the probability density is the greatest. Any point on the wave gives the same trajectory up to translation.) The magnetic translation operator is the usual translation operator times a gauge transformation so that it commutes with the Landau Hamiltonian [10–12]. It has the same effect as the usual translation operator in shifting the wave's probability distribution. But magnetic translation in two different directions do not commute, and when the wave is brought back to its initial position, it acquires a phase factor $\exp(-iq\phi_B/\hbar c)$, where ϕ_B is the magnetic flux through the loop.

The above is a kinematical argument based on the magnetic translation symmetry of the Landau Hamiltonian. However, such a magnetic translation around a loop can be implemented by the Schrödinger time evolution, if a uniform in-plane electric field is applied that changes direction with time [13]. The Berry phase factor accompanying such an evolution is $\exp(-iq\phi_B/\hbar c)$. So now the phase factor has physical consequences if an interference experiment is performed. Observe that there is a difference between the phase factor here and the Aharonov–Bohm phase factor $\exp(iq\phi/\hbar c)$, which has also been derived from a Berry phase approach [3] in the original context where there is no magnetic field outside a flux line. One can imagine driving a wave packet around a loop that encircles both a uniform magnetic field and a flux line whose magnetic field points in the same direction. Then ϕ and ϕ_B have the same sign but the final geometric phase would be $\frac{q}{\hbar c}(\phi-\phi_B)$ rather than $\frac{q}{\hbar c}(\phi+\phi_B)$. However, a rigorous theoretical description of such a process is complicated by the fact that the flux line is mixed with the uniform magnetic field so that the magnetic translation symmetry of the original Landau problem does not exist any more.

In addition to the sign difference in $\frac{q\phi}{\hbar c}$ and $-\frac{q\phi_B}{\hbar c}$, ϕ_B is different from ϕ in that it is not located in a region separated from the quantum wave. At each instant during the evolution, the wave is in

[1] This point was discussed by Wu and Yang [8] in connection with the Aharonov–Bohm effect. Although in their discussions the line of monopoles is absent, their arguments carry over to the present situation without essential change.