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Generalized entanglement constraints in multi-qubit systems in terms of Tsallis entropy

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ABSTRACT

We provide generalized entanglement constraints in multi-qubit systems in terms of Tsallis entropy. Using quantum Tsallis entropy of order q, we first provide a generalized monogamy inequality of multi-qubit entanglement for $q = 2$ or 3. This generalization encapsulates the multi-qubit CKW-type inequality as a special case. We further provide a generalized polygamy inequality of multi-qubit entanglement in terms of Tsallis-q entropy for $1 \leq q \leq 2$ or $3 \leq q \leq 4$, which also contains the multi-qubit polygamy inequality as a special case.

1. Introduction

Quantum Tsallis entropy is a one-parameter generalization of von Neumann entropy with respect to a nonnegative real parameter q [1,2]. Tsallis entropy is used in many areas of quantum information theory such as the characterization of classical statistical correlations inherent in quantum states [3], and some conditions for separability of quantum states [4–6]. There are also discussions about using the non-extensive statistical mechanics to describe quantum entanglement in terms of Tsallis entropy [7].

As a function defined on the set of density matrices, Tsallis entropy is concave for all $q > 0$, which plays an important role in quantum entanglement theory. Because the concavity of Tsallis entropy assures the property of *entanglement monotone* [8], it can be used to construct a faithful entanglement measure, which does not increase under *local quantum operations and classical communication* (LOCC).

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One distinct property of quantum entanglement from other classical correlations is that multi-party entanglement cannot be freely shared among the parties. This restricted shareability of entanglement in multi-party quantum systems is known as *monogamy of entanglement* (MoE) [9,10]. MoE is a key ingredient for secure quantum cryptography [11,12], and it also plays an important role in condensed-matter physics such as the N-representability problem for fermions [13].

Using *concurrence* [14] as a bipartite entanglement measure, Coffman–Kundu–Wootters (CKW) provided a mathematical characterization of MoE in three-qubit systems as an inequality [15], which was generalized for arbitrary multi-qubit systems [16]. As a dual concept of MoE, a *polygamy* inequality of multi-qubit entanglement was established in terms of *concurrence of assistance* (CoA). Later, it was shown that the monogamy and polygamy inequalities of multi-qubit entanglement can also be established by using other entropy-based entanglement measures such as Rényi, Tsallis and unified entropies [17–19].

Recently, a different kind of monogamous relation in multi-qubit entanglement was proposed by using concurrence and CoA [20]. Whereas the CKW-type monogamy inequalities of multi-qubit entanglement provide a lower bound of bipartite entanglement between one qubit subsystem and the rest qubits in terms of two-qubit entanglement, this new kind of monogamy relations in [20] provides the bounds of bipartite entanglement between a two-qubit subsystem and the rest in multi-qubit systems in terms of two-qubit concurrence and CoA.

Here, we provide generalized entanglement constraints in multi-qubit systems in terms of Tsallis entropy for a selective choice of the real parameter q. Using quantum Tsallis entropy of order q, namely *Tsallis-q entropy*, we first show that the CKW-type monogamy inequality of multi-qubit entanglement can have a generalized form for $q = 2$ or 3. This generalized monogamy inequality encapsulates multi-qubit CKW-type monogamy inequality as a special case. We further provide a generalized polygamy inequality of multi-qubit entanglement in terms of Tsallis-q entropy for $1 \leq q \leq 2$ or $3 \leq q \leq 4$, which also contains multi-qubit polygamy inequality as a special case.

This paper is organized as follows. In Section 2.1, we recall the definition of Tsallis-q entropy, and the bipartite entanglement measure based on Tsallis entropy, namely Tsallis-q entanglement as well as its dual quantity, Tsallis-q entanglement of assistance (TEoA). In Section 2.2, we review the analytic evaluations of Tsallis-q entanglement and TEoA in two-qubit systems based on their functional relations with concurrence, and we further review the monogamy and polygamy inequalities of multi-qubit entanglement in terms of Tsallis-q entanglement and TEoA in Section 3. In Section 4, we provide generalized monogamy and polygamy inequalities of multi-qubit entanglement in terms of Tsallis-q entanglement and TEoA, and we summarize our results in Section 5.

2. Tsallis-*q* entanglement

2.1. Definition

Using a generalized logarithmic function with respect to the parameter q,

$$\ln_q x = \frac{x^{1-q} - 1}{1 - q}, \tag{1}$$

quantum Tsallis-q entropy for a quantum state ρ is defined as

$$S_q(\rho) = -\mathrm{tr}\rho^q \ln_q \rho = \frac{1 - \mathrm{tr}(\rho^q)}{q - 1} \tag{2}$$

for $q > 0,\ q \neq 1$ [2]. Although the quantum Tsallis-q entropy has a singularity at $q = 1$, it converges to von Neumann entropy when q tends to 1 [21],

$$\lim_{q\to 1} S_q(\rho) = -\mathrm{tr}\rho \ln \rho = S(\rho). \tag{3}$$

Based on Tsallis-q entropy, a class of bipartite entanglement measures was introduced; for a bipartite pure state $|\psi\rangle_{AB}$ and each $q > 0$, its *Tsallis-q entanglement* [18] is

$$\mathcal{T}_q\left(|\psi\rangle_{A|B}\right) = S_q(\rho_A), \tag{4}$$