Annals of Physics 373 (2016) 298–324

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Wigner transformation, momentum space topology, and anomalous transport

M.A. Zubkov [*,1]

LE STUDIUM, Loire Valley Institute for Advanced Studies, Tours and Orleans, 45000 Orleans, France
Laboratoire de Mathématiques et de Physique Théorique, Université de Tours, 37200 Tours, France
ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia
National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoe highway 31, 115409 Moscow, Russia

ARTICLE INFO

Article history:
Received 5 April 2016
Accepted 13 July 2016
Available online 20 July 2016

Keywords:
Anomalous quantum Hall effect
Chiral magnetic effect
Topological insulators
Weyl semimetals
Wigner transformation
Momentum space topology

ABSTRACT

Using derivative expansion applied to the Wigner transform of the two- point Green function we analyse the anomalous quantum Hall effect (AQHE), and the chiral magnetic effect (CME). The corresponding currents are proportional to the momentum space topological invariants. We reproduce the conventional expression for the Hall conductivity in $2 + 1$ D. In $3 + 1$ D our analysis allows to explain systematically the AQHE in topological insulators and Weyl semimetals. At the same time using this method it may be proved, that the equilibrium CME is absent in the wide class of solids, as well as in the properly regularized relativistic quantum field theory.

1. Introduction

Momentum space topology is becoming the important tool for the study of the ground states of condensed matter systems (for the review see [1–5]). In particular, the momentum space topological invariants protect gapless fermions on the boundaries of topological insulators [6,7]. Topological

* Correspondence to: ITEP, B. Cheremushkinskaya 25, Moscow, 117259, Russia.
E-mail address: zubkov@itep.ru.

1 On leave of absence from Moscow Institute of Physics and Technology, 9, Institutskii per., Dolgoprudny, Moscow Region, 141700, Russia.

http://dx.doi.org/10.1016/j.aop.2016.07.011

invariants in momentum space protect also the bulk gapless fermions in Dirac and Weyl semimetals [8,9]. The large variety of topological defects and textures exist in the fermionic superfluids, and the gapless fermions associated with these objects are described by momentum space topology [10]. Momentum space topology was also discussed in the context of relativistic quantum field theory (QFT) [11–15,8,16–18]. In [19] the topological invariants in momentum space have been considered for the lattice regularization of QFT with Wilson fermions. Appearance of the massless fermions at the intermediate values of bare mass parameter was related to the jump of the introduced momentum space topological invariant. This invariant may actually be used for the description of a certain class of topological insulators.[2] In [20] the model with overlap fermions has been considered on the same grounds. In particular, the possible physical meaning of the zeros of the Green function has been discussed. The appearance of zeros of the Green function, in turn, has been discussed in the context of condensed matter physics (see, for example, [21]).

The momentum space topological invariants are expressed in terms of the Green functions. Therefore, they are applicable both to the non-interacting and to the interacting systems [8]. Suppose, that we start from the model without interactions. When the interactions are turned on, the value of the topological invariant is not changed until the phase transition is encountered. This means, that the properties of the system described by the given topological invariant are robust to the introduction of interactions. The more simple non-interacting model may be investigated in order to describe such properties of the complicated interacting system. In the present paper we apply momentum space topology to the description of the anomalous quantum Hall effect (AQHE) in topological insulators and Weyl semimetals. In [22] we considered the chiral magnetic effect (CME) (mainly, in the framework of relativistic quantum field theory) using the approach based on momentum space topology. Here we briefly repeat our proof of the absence of the equilibrium bulk CME with the emphasis in the application to the solid state systems.

Actually, momentum space topology with the topological invariants expressed through the Green functions represents the alternative to the less powerful but more popular technique of Berry curvature proposed to describe the QHE in [23] and developed later in a number of publications (see, for example, [24] and references therein). The technique of Berry curvature may be applied to the noninteracting condensed matter systems with Green function of the form $\mathcal{G}^{-1} = i\omega - \hat{H}$, where ω is the imaginary frequency while $\hat{H}$ is the Hamiltonian. Unfortunately, this formalism does not allow to deal in a similar way with the interacting systems with more complicated dependence of the Green function on the imaginary frequency. Besides, in spite of all its advantages, the Berry curvature formalism does not allow to describe the response of the system to the external magnetic (rather than electric) field. Finally, the bulk-boundary correspondence remains out of this formalism. Those three points are improved in the technique that utilizes the topological invariants composed directly of the Green functions. It was proposed first by G.E.Volovik (see [8,6] and references therein). Following [22] in the present paper we develop this technique and give the relation between the electromagnetic response of electric current and the topological invariants in momentum space of both 2 + 1D and 3 + 1D systems. In the present paper we emphasize, that those topological invariants are constructed of the Wigner transform of the two-point Green functions, which allows to apply the proposed methodology to the direct description of bulk-boundary correspondence. As for the Berry curvature formalism, we demonstrate that it follows from our approach as a particular case.

The family of the non-dissipative transport effects related to chiral anomaly has been widely discussed recently both in the context of the high energy physics and in the context of condensed matter theory [25–28,53,29–32,32–34]. In particular, the possible appearance of such effects in the recently discovered Dirac and Weyl semimetals has been considered [54–58,35,36]. Besides, the possibility to observe those effects in relativistic heavy-ion collisions was proposed [37,38]. The chiral magnetic effect (CME) is the generation of electric current in the presence of external magnetic field and chiral chemical potential [39,40]. The quantum Hall effect in the 2 + 1D and the 3 + 1D

[2] This pattern is complimentary to that of [7], where in the similar way the number of boundary gapless fermions is related to the jump of the topological invariant across the boundary in coordinate space.