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Dissipative quantum trajectories in complex space: Damped harmonic oscillator

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ABSTRACT

Dissipative quantum trajectories in complex space are investigated in the framework of the logarithmic nonlinear Schrödinger equation. The logarithmic nonlinear Schrödinger equation provides a phenomenological description for dissipative quantum systems. Substituting the wave function expressed in terms of the complex action into the complex-extended logarithmic nonlinear Schrödinger equation, we derive the complex quantum Hamilton–Jacobi equation including the dissipative potential. It is shown that dissipative quantum trajectories satisfy a quantum Newtonian equation of motion in complex space with a friction force. Exact dissipative complex quantum trajectories are analyzed for the wave and solitonlike solutions to the logarithmic nonlinear Schrödinger equation for the damped harmonic oscillator. These trajectories converge to the equilibrium position as time evolves. It is indicated that dissipative complex quantum trajectories for the wave and solitonlike solutions are identical to dissipative complex classical trajectories for the damped harmonic oscillator. This study develops a theoretical framework for dissipative quantum trajectories in complex space.

1. Introduction

Various trajectory methods, based on classical, semiclassical, or quantum mechanics, have been developed to study dynamical problems in physics and chemistry [1–4]. For example, in the framework of Bohmian mechanics [5,6], the quantum trajectory method (QTM) provides nontraditional

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computational techniques for solving the time-dependent Schrödinger equation (TDSE) [7–9]. In contrast with the QTM in real space, the complex QTM, based on the quantum Hamilton–Jacobi formalism [10,11], has been developed to provide not only insightful interpretation but also computational utility for quantum dynamical processes. This method has been used to analyze stationary state problems [12–18], quantum interference [19–22], and coherent states [17,23]. The complex probability density and flux continuity have been discussed in detail [24–29]. As a computational tool, the complex QTM involving propagation of approximate independent quantum trajectories in complex space has been applied to wave packet scattering problems [30–34], dissipative dynamics [35], nonadiabatic dynamics [36,37], and the semiclassical coherent state propagator [38]. The path-integral derivation has been presented for the complex QTM and the complex WKB method [39]. In addition, the dynamics of tunneling through barriers has been studied using complex time paths [40,41]. Furthermore, we have proposed a computational method to integrate the complex quantum Hamilton–Jacobi equation (CQHJE) by propagating an ensemble of correlated Bohmian trajectories in real space [42–44].

Several studies have involved propagation of quantum trajectories with friction. In order to improve the accuracy and stability of the QTM, artificial viscosity terms were added to the equations of motion to soften the quantum force and to prevent nodes from fully forming [45–51]. The ground state of quantum systems can be realized from an initial nonstationary state by adding a friction term to the Newtonian-type equation of motion of the QTM [52–55]. The friction method has been proposed to stabilize the numerical implementation of the quantum trajectory formulation [56,57]. In particular, the dissipative dynamics associated with the Caldirola–Kanai time-dependent Hamiltonian model has been analyzed in the framework of Bohmian mechanics [58]. However, these studies are associated with the quantum trajectory evolution with friction in real space.

The purposes of the current study are to develop the theoretical formulation of dissipative quantum trajectories in complex space and to analyze these trajectories for the damped quantum harmonic oscillator. For comparison, we will begin with briefly reviewing Kostin's Schrödinger–Langevin equation providing a phenomenological description for dissipative quantum systems [59,60]. The Schrödinger–Langevin equation has been discussed and analyzed for several quantum systems, such as the damped harmonic oscillator and the motion of a charged particle in the presence of damping [3,61–63]. Several methods have been developed to obtain solutions to the nonlinear Schrödinger–Langevin equation using quantum fluid dynamics [64–67]. The nonlinear Schrödinger–Langevin equation has been generalized for quantum processes in the presence of nonlinear friction and a heat bath [68], and a non-Markovian nonlinear Schrödinger–Langevin equation has been derived from the system-plus-bath approach [69]. In addition, the ground states of several one-dimensional quantum systems have been obtained through the fixed-grid integration of the Schrödinger–Langevin equation [70]. Furthermore, a nonlinear logarithmic Schrödinger equation under continuous measurement has been proposed [71], and the establishment of a dividing line between the classical and quantum regimes is one of the main aspects of the measurement process [72]. Recently, the Schrödinger–Langevin equation has been studied with thermal fluctuations by including the stochastic force term [73].

As in Bohmian mechanics, substituting the polar form of the wave function into the Schrödinger–Langevin equation, we obtain the continuity equation for the probability density and the modified quantum Hamilton–Jacobi equation (QHJE) with the dissipative potential depending on the real action function. These dissipative Bohmian trajectories obey a quantum Newtonian equation of motion including a friction force. As time evolves, Bohmian particles continue to lose the kinetic energy, and then these particles become stationary when equilibrium is reached. Based on the hydrodynamic formulation, the Schrödinger–Langevin equation provides an insightful interpretation for dissipative Bohmian trajectories in real space. Recently, we have integrated the Schrödinger–Langevin equation for the ground state of quantum systems by evolving an ensemble of correlated Bohmian trajectories [74]. In addition, the Schrödinger–Langevin equation has been approximately solved for the ground state energy of quantum systems by propagating one single trajectory at a fixed point [75].

However, the Schrödinger–Langevin equation has several unexpected features [62,76–78]. For example, the solutions for a damped harmonic oscillator contain the undamped frequency instead of the reduced frequency. In addition, the probability density satisfies the reversible continuity equation for a system displaying damping which follows an irreversible dynamics. In order to avoid the