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Quantum teleportation and entanglement swapping of electron spins in superconducting hybrid structures

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ABSTRACT

We present schemes for quantum teleportation and entanglement swapping of electronic spin states in hybrid superconductor–normal-metal systems. The proposed schemes employ subgap transport whereby the lowest order processes involve Cooper pair-electron and double Cooper-pair cotunneling in quantum teleportation and entanglement swapping protocols, respectively. The competition between elastic cotunneling and Cooper-pair splitting results in the success probability of 25% in both cases. Described implementations of these protocols are within reach of present-day experimental techniques.

1. Introduction

Quantum teleportation is one of the major paradigms of a rapidly advancing field of quantum information science. As described in the original proposal [1], an unknown state of a qubit can be disembodied into classical and quantum parts, and subsequently a perfect replica qubit reconstructed at a distant location without transfer of matter through the intervening space. Although the quantum part is transferred instantly, the teleportation protocol is in agreement with special relativity since the information transfer rate is limited by the classical channel. The first experimental validation of quantum teleportation employed photons [2]. Subsequently teleportation has been demonstrated in a variety of systems where information was transferred between two individual material particles, such as trapped ions [3,4] or nuclei [5], as well as between light and matter [6]. Here we propose a novel

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implementation of the teleportation protocol, whereby the spin of an electron tunneling through one junction of an electronic circuit is teleported onto the spin of another electron tunneling through a distant junction of the same circuit, with superconducting electrodes acting as a source and a joint state measurement apparatus for the entangled electron pairs.

Explorations of entanglement in solid state systems are of importance for fundamental studies of quantum mechanics, such as tests of Bell's inequalities [7], and two-particle interference in fermionic Hanbury Brown and Twiss-type experiments [8]. A variety of physical mechanisms have been considered as a basis for entanglement production, for instance pairing interaction in a superconductor [9–15], Kondo scattering by a magnetic impurity [16], or Coulomb interaction in a quantum dot [17–19]. Whereas photon entanglement can be generated by means of linear optics using a beam splitter [20], it cannot be generated from sources in thermal equilibrium [21]. In contrast, the existence of the Fermi sea permits entanglement of electrons without interactions. Any two-channel conductor can be used to entangle the left and the right outgoing states from a localized scatterer. In particular, it was shown in [22] that edge channel transport in the integer quantum Hall effect can be used as a source of entangled electron–hole pairs. Entanglement sources based on quantum dots [23,24], and a two-dimensional electron gas in a high magnetic field [25], have been suggested for implementation of teleportation protocols. Although photon-mediated teleportation appears an obvious choice for applications such as long distance communication [26,27], it is of interest to find alternatives for short distance communication (such as between adjacent quantum processors). Furthermore, teleportation could be harnessed as a resource to perform universal quantum computation [28], as well as error correction in fault-tolerant quantum computation [29]. Therefore, it is worth noting that electronic systems have obvious advantage over photons in terms of integration.

Quantum teleportation is based on the following identity for any qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$:

$$|\psi\rangle \otimes |\beta_{zx}\rangle = \frac{1}{2}\sum_{z',x'} |\beta_{z'x'}\rangle \otimes (-1)^{zx'} X^{x+x'} Z^{z+z'} |\psi\rangle, \tag{1}$$

where $|\beta_{zx}\rangle = [|0, x\rangle + (-1)^z |1, \bar{x}\rangle]/\sqrt{2},\ z, x \in \{0, 1\}$, are the Bell states, and X, Z are the Pauli matrices. Transmitting the state $|\psi\rangle$ requires splitting an Einstein–Podolsky–Rosen (EPR) pair ($|\beta_{zx}\rangle$) [30] between the sender and the receiver, with the sender subsequently performing the joint measurement on the two qubits in her possession, ($|\beta_{z',x'}\rangle$), and sending to the receiver the classical bits (z, x) and (z', x') (or, if this information has been shared beforehand, just sending the one-bit signal that the joint measurement has been performed). After the procedure, only the receiver is in possession of the state $|\psi\rangle$, because the joint measurement destroyed the copy at the sender's end, in agreement with the no cloning theorem [31]. In our implementation, described in detail in the following section, the physical representation of the qubit is the electron spin ($|\psi\rangle \equiv |\sigma\rangle, |0\rangle \equiv |\uparrow\rangle, |1\rangle \equiv |\downarrow\rangle$); the original EPR pair is in the singlet state, as is the pair in the sender's possession after their joint measurement, i.e. $(z, x) = (z', x') = (1, 1)$. It follows from Eq. (1) that in this case the receiver does not need to perform any rotation of his qubit state, because after the joint measurement by the sender, the receiver's qubit is automatically projected onto the state $|\psi\rangle$ (disregarding the irrelevant minus sign). Fixing one-out of four possible results of the Bell state measurement decreases efficiency, but does not reduce fidelity [2].

Teleportation can be applied not only to pure states, but also to mixed states. The entanglement swapping protocol [32,33] is one such procedure whereby two pairs of entangled particles (0–1 and 2–3) are produced by different sources, and the subsequent Bell state measurement is performed on the 1–2 pair, resulting in entanglement of the 0–3 pair. For example, when both pairs are generated in the antisymmetric states, we can write:

$$|\beta_{11}\rangle_{01} \otimes |\beta_{11}\rangle_{23} = \frac{1}{2}\sum_{z,x} (-1)^{z+x+1} |\beta_{zx}\rangle_{03} \otimes |\beta_{zx}\rangle_{12}. \tag{2}$$

By performing the joint measurement of the pair 1–2 ($|\beta_{zx}\rangle_{12}$), the pair 0–3 is projected onto the same state ($|\beta_{zx}\rangle_{03}$); and therefore entanglement has been swapped from the pairs 0–1 and 2–3, to the pairs 0–3 and 1–2. In this way, particles can be entangled not only when they do not share any common past (i.e. without interacting and emerging from different sources), but also *a posteriori*