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Incorporation of generalized uncertainty principle into Lifshitz field theories

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ABSTRACT

In this paper, we will incorporate the generalized uncertainty principle into field theories with Lifshitz scaling. We will first construct both bosonic and fermionic theories with Lifshitz scaling based on generalized uncertainty principle. After that we will incorporate the generalized uncertainty principle into a non-abelian gauge theory with Lifshitz scaling. We will observe that even though the action for this theory is non-local, it is invariant under local gauge transformations. We will also perform the stochastic quantization of this Lifshitz fermionic theory based generalized uncertainty principle.

1. Introduction

The classical picture of spacetime breaks down in most approaches to quantum gravity. This is due to the fluctuations in the geometry being of order one at Planck scale. Thus, the picture of spacetime as a continuous differential manifold cannot be valid below Planck length. Furthermore, the existence of a minimum length scale is also a feature of string theory [1–5]. In fact, in loop quantum gravity the existence of minimum length turns big bang into a big bounce [6]. However, the existence of minimum length is not consistent with conventional uncertainty principle, which states that one can measure length with arbitrary accuracy, if one takes no measurement of momentum [1,2,7–19]. Thus, the uncertainty principle has to be modified if one wants to incorporate the existence of minimum length scale. These considerations have led to a modification of the Heisenberg uncertainty principle,

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which in turn has led to a modification of the Heisenberg algebra. It may be noted that the implications of this modified uncertainty principle for quantum field theory have also been studied [20–22]. In this paper, we analyse a quantum field theory based on generalized uncertainty with Lifshitz scaling. Lifshitz field theories are quantum field theories based on an anisotropic scaling between space and time.

Lifshitz theories were first introduced in condensed matter physics to model quantum criticality [23–26]. In fact, a Fermi-surface-changing Lifshitz transition occurs for some heavy fermion compounds [27]. The location of this Fermi-surface-changing Lifshitz transition is influenced by carrier doping. Due to strong correlations, a heavy band does not shift rigidly with the chemical potential and the actual shift is determined by the interplay of heavy and additional light bands crossing the Fermi level. Furthermore, meta-magnetic transitions in models for heavy fermions have also been analysed using doped Kondo lattice model in two dimensions [28]. Some heavy fermion metals display a field-driven quantum phase transition due to a breakdown of the Kondo effect [29,30]. Many of the properties have been described by a Zeeman-driven Lifshitz transition of narrow heavy fermion bands [31]. Materials that cannot be described with the local dielectric response have been described by a generalization of the usual Lifshitz theory [32]. In fact, the temperature correction to the Casimir–Lifshitz free energy between two parallel plates made of dielectric material, possessing a constant conductivity at low temperatures, has been calculated [33]. Lifshitz theory has also been used for calculating the van der Waals and Casimir interaction between graphene and a material plate, graphene and an atom or a molecule, and between a single-wall carbon nanotube and a plate [34]. In this model the reflection properties of electromagnetic oscillations on graphene are governed by the specific boundary conditions imposed on the infinitely thin positively charged plasma sheet, carrying a continuous fluid with some mass and charge density.

Fermionic retarded Green's function with $z = 2$ has been studied at finite temperature and finite chemical potential [35]. Here the usual Lifshitz geometry was replaced by a Lifshitz black hole. Hawking radiation for Lifshitz fermions has also been studied [36]. Fermionic theories with $z = 2$ Lifshitz scaling have also been constructed using a non-local differential operator [37]. This non-local differential operator is defined using harmonic extension of a compactly supported function [38–42]. It appears as a map from the Dirichlet-type problem to the Neumann type problem. It may be noted that fermionic theories with $z = 3$ have also been studied [43,44]. It has been demonstrated that Nambu–Jona-Lasinio type four-fermion coupling at the $z = 3$ Lifshitz fixed point in four dimensions is asymptotically free and generates a mass scale [45]. In this paper, we will study both bosonic and fermionic Lifshitz field theory, consistent with generalized uncertainty principle. We will also study the gauge symmetry for these theories. It may also be noted that another interesting deformation of quantum mechanics comes from stochastic quantization [46–49]. Stochastic quantization has provided a powerful framework for analysing bosonic theories with Lifshitz scaling [50,51]. In fact, effect of ohmic noise on the non-Markovian spin dynamics resulting in Kondo-type correlations has been studied using stochastic quantization [52] In this paper, we will analyse the stochastic quantization of Lifshitz Dirac equation with minimum length.

2. Generalized Uncertainty Principle

In the Lifshitz field theories the scaling is usually taken as $x \to bx$ and $t \to b^z t$, where b is called the scaling factor and z is called the degree of anisotropy. For $z = 1$, this reduces to the usual conformal transformation. In this paper, we will analyse the Lifshitz theories with $z = 2$. The Lifshitz action for a bosonic field with $z = 2$, can be written as [37]

$$S_b = \frac{1}{2}\int d^{d+1}x\, (\phi\partial^0\partial_0\phi - \kappa^2\phi(\partial^i\partial_i)^2\phi). \tag{1}$$

The Lifshitz theories are unitarity because they contain no higher order temporal derivatives. So, we will leave the temporal part of the Lifshitz action for a bosonic field undeformed. However, we will deform its spatial part, to make it consistent with the existence of a minimum measurable length [19,20]. The Heisenberg uncertainty principle is not consistent with the existence of a