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The effect of quenched bond disorder on first-order phase transitions

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We investigate the effect of quenched bond disorder on the two-dimensional three-color Ashkin–Teller model, which undergoes a first-order phase transition in the absence of impurities. This is one of the simplest and striking models in which quantitative numerical simulations can be carried out to investigate emergent criticality due to disorder rounding of first-order transition. Utilizing extensive cluster Monte Carlo simulations on large lattice sizes of up to 128×128 spins, each of which is represented by three colors taking values ± 1, we show that the rounding of the first-order phase transition is an emergent criticality. We further calculate the correlation length critical exponent, ν, and the magnetization critical exponent, β, from finite size scaling analysis. We find that the critical exponents, ν and β, change as the strength of disorder or the four-spin coupling varies, and we show that the critical exponents appear not to be in the Ising universality class. We know of no analytical approaches that can explain our non-perturbative results. However our results should inspire further work on this important problem, either numerical or analytical.

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1. Introduction

Disorder is an inevitable part of any condensed matter system and therefore its study has always been of great importance. The effect of randomness on the transition temperature (random T_c) and randomness coupled to the site variables on continuous phase transitions have been extensively studied for a long time, and they appear to be well understood.

For random T_c model defined above, the Harris criterion [1] predicts conditions under which disorder can change the universality class of the pure system. He showed that if the specific heat exponent, α, of a pure system is positive, i.e., if the product of the correlation length critical exponent, ν, and the dimension of the system, D, is less than 2 ($D\nu < 2$), the effect of impurity is relevant: the pure system's fixed point is unstable, the system's critical exponents change, and the disordered system does not remain in the universality class of the pure system. On the other hand, if the specific heat critical exponent, α, of a pure system is negative, i.e., if $D\nu > 2$, the effect of impurity is irrelevant: the pure system's fixed point is stable, critical exponents do not change, and the disordered system remains in the universality class of the pure system. If $\alpha = 0$, the situation is marginal.

The problem of random field coupled linearly to the order parameter is different, however. Using the domain-wall argument, Imry and Ma [2] showed that, for a D dimensional system with discrete order parameter, the stability condition of order requires $D/2 \leq D - 1$. For the continuous case the corresponding stability condition is $D/2 \leq D - 2$. The marginal cases are treated in Refs. [3,4]. The increase of critical dimensionality is verified for the random field Ising model in Ref. [5]; for a review of this model, see Ref. [6].

First-order transitions are ubiquitous in both classical and quantum systems, because they do not require any fine tuning of the coupling constant. However, in contrast to the effect of disorder on continuous transitions much less is known about its effect on first-order transitions. Rounding of first-order phase transition due to quenched impurities that couple to energy-like variables has been studied in the past, yet the results are still not fully elucidated. In an early study, Imry and Wortis [7], made use of Imry–Ma [2] domain-wall argument and showed that the presence of quenched bond randomness may produce rounding of a first-order phase transition. This happens because bond randomness couples to the local energy density of the system the same way that the random field couples to the local magnetization. Using a renormalization-group calculation, Hui and Berker [8,9] confirmed this idea and showed that, for the q-states Potts model, the bond randomness turns a first-order phase transition into a second order transition. In another work, Aizenman and Wehr [3,4] rigorously proved the elimination of discontinuity in the density of the variable conjugate to the fluctuating order parameter. Specifically, they showed the absence of the latent heat for the q-state random bond Potts model. The effect of quenched bond randomness on quantum systems that undergo a first-order phase transition in the pure case has also been touched upon, but without any firm conclusions involving the nature of the criticality and critical exponents [10–14].

We list further studies [15–19] in two-dimensions (2D). The questions to be answered are whether or not the rounding is an emergence of criticality, i.e., does the correlation length diverge? If so, what are the exponents and what are the universality classes, if any? The $q = 8$ state random bond Potts model [15], which has first order transition in the pure system, hinted on the consistency with the universality class of the pure 2D Ising model, which is known to have $\nu = 1$ and $\beta = 1/8$. However, in a later study of the critical behavior of the random bond Potts model it was found [17] that although the correlation length critical exponent ν is numerically close to unity (Ising), the magnetic exponent β/ν is far from the value 1/8 and varies continuously with q, and therefore the disordered system cannot be in the universality class of the pure Ising model [20–22]. Similar behavior was also observed in the study by Chatelain and Berche [23].

As to one [18] and two-loop [19] *perturbative renormalization group calculations*, N-color AT model hinted at the Ising universality class, contrary to our present work, as well as our recent work in smaller lattices, 32×32 [24]. The validity of such perturbative calculations can of course be doubted, as the renormalization group trajectories flow to strong coupling before curling back to the pure N-decoupled Ising fixed points.

Our previous work [24] could also be doubted as to whether or not the observed behavior is an artifact of finite size effects. To address this issue, it is important to do the calculations on larger system