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Two qubit entanglement preservation through the addition of qubits

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ABSTRACT

An entanglement preservation scheme is proposed by considering the exact evolution of an N-qubit interacting system in a common reservoir. We find that the steady-state concurrence is dependent only on the number of qubits, the qubit–reservoir coupling strength and the initial conditions of the system. Furthermore, we show that as $N \to \infty$, the initial entanglement between the two qubits is preserved.

1. Introduction

Entanglement is a fundamental property of quantum systems and serves as a resource in various quantum information processes. Some of its pioneering applications include quantum key distribution, quantum dense coding, quantum teleportation and quantum computation [1]. However it is so fragile and undergoes either an asymptotic decay or a sudden death [2–8]. This is due to decoherence, whereby the unavoidable interactions of the entangled system with its environments alter the quantum system, losing entanglement in the process [9,10]. In fact, the fragility of entanglement is the main reason recent researchers turned to quantum discord (another form of quantum correlation) as a resource for quantum computation [10,11].

Although it is known that there exist so-called decoherence-free (also known as subradiant) states whose initial entanglement is invariant over time when a bipartite system interacts with a common environment [12–14], any other initially entangled state not found within this decoherence-free subspace will inevitably lose its entanglement. Hence, a number of papers have been done regarding entanglement preservation and these could be classified under variants of two schemes [15], one which

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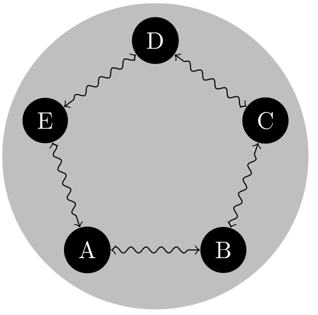

Fig. 1. The figure corresponds to five qubits (black circles) immersed in a common zero-temperature thermal reservoir (gray circle), each qubit interacting with its two neighbors.

employs the quantum Zeno effect [12,16–18] and the other which uses detuning modulation [15,19]. The former scheme is relatively difficult since one has to perform a series of measurements to the system during the course of the evolution while for the latter, entanglement is not preserved in the long-time limit. In this Letter, we propose a scheme without the disadvantages of the aforementioned schemes. It involves preparing entangled qubits and letting them evolve with other qubits immersed in a common environment. We show that simply making the number of qubits large enough preserves entanglement whether or not the initial states are decoherence-free.

The protection of the long-time limit of entanglement between two qubits via the addition of a qubit was first observed in [14] where they concluded that the bipartite entanglement is sustained by the addition of a third qubit while it is lost completely when the two qubits evolve alone without the third. We find in this paper that the protection provided by the addition of qubits is actually dependent on the initial conditions. The two-qubit entanglement in the presence of N qubits was also considered in [20] with the conclusion that the entanglement decreases as N increases for initially unentangled qubits. We show that this observation is a special case of our results here in a sense that if we consider the two-qubits to be initially entangled, we see the exact opposite result, that is, increasing N preserves the long-time entanglement. Now, to test whether the entanglement preservation described above is robust against different spectral densities, we consider numerically the cases for Ohmic, sub-Ohmic, and super-Ohmic spectral densities and compare the entanglement evolution with the exact solution we obtain using a Lorentzian spectral density. We will find that indeed, entanglement is still preserved in the long-time limit for all spectral densities for large N.

The paper is arranged as follows. In Section 2, we introduce the model to be used and derive its exact evolution. Then in Section 3, we analyze the entanglement dynamics between two qubits and how it is affected by the different properties of the reservoir and the presence of the other $N - 2$ qubits. Section 4 shows numerically the evolution for Ohmic, sub-Ohmic and super-Ohmic spectral densities. Finally, we conclude in Section 5.

2. The model and its evolution

To achieve our scheme of entanglement preservation, we study the dynamics of an N-qubit system immersed in a common zero-temperature thermal reservoir. We utilize the model with an Ising-type coupling since aside from fact that it is exactly solvable, we want to observe the effects of the qubit–qubit interaction. We will then be able to see the effects of an arbitrary number of qubits on the entanglement dynamics between two qubits and show that as $N \to \infty$, the steady-state entanglement between two qubits approaches the initial entanglement. Here, we will assume that the strength of all the qubit–qubit interactions is the same which allows us to obtain an exact analytical solution without Born and Markovian approximations. The entanglement dynamics between two arbitrary qubits will be analyzed using Wootters' concurrence [21].

We now consider N qubits in an Ising chain, each qubit interacting with its neighbors via the Ising-type coupling. For this model (see Fig. 1 for the case of 5 qubits), we suppose that all qubits are coupled