Annals of Physics 354 (2015) 31–71

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Longitudinal oscillations in a non-uniform spatially dispersive plasma

Alex Calogeracos

Physics Division, HAFA, Dekelia Attikis, Greece

ARTICLE INFO

Article history:
Received 22 August 2014
Accepted 9 December 2014
Available online 16 December 2014

Keywords:
Spatial dispersion
Surface plasmons
Levinson's theorem

ABSTRACT

Longitudinal oscillations of the electron fluid in the hydrodynamic model of a metal are examined with pressure effects taken into account. It is well-known that this entails spatial dispersion. The equilibrium electron number density is taken to be non-uniform and a non-self-adjoint fourth order differential equation obeyed by the electric potential is derived. A velocity potential necessary for the description of sound waves is introduced in the standard fashion and the generalized version of Bloch orthogonality appropriate to a non-uniform background is deduced. We observe a duality between electric and velocity potentials in the sense that the respective differential operators are adjoint to each other. The spectrum is calculated in the special case of an exponential profile for the equilibrium electron number density. The surface plasmons are connected with the analytic properties of the scattering amplitude in the complex plane. The phase shift at threshold is expressed in terms of the number of surface plasmon modes via an expression reminiscent of Levinson's statement in quantum mechanics.

1. Introduction

The hydrodynamic model of a metal was introduced by Bloch [1,2] and elaborated upon by Jensen [3]. An older review with theoretical applications is offered by Barton [4] and a more recent one containing the exciting developments of the past few decades by Pitarke et al. [5]. The existence of surface plasmon modes is one important prediction of any model dealing with an air–metal interface, and in fact the review just mentioned is wholly devoted to this issue. Bennett [6] was probably the first to

E-mail addresses: a.calogeracos@yahoo.co.uk, acalogeracos.hafa@haf.gr.

http://dx.doi.org/10.1016/j.aop.2014.12.005
0003-4916/

consider an air–metal interface and the seminal paper by Eguiluz et al. [7] came soon after. These last two papers pointed out the existence of multipole surface plasmons in non-uniform metals (i.e. surface plasmons further to the one described in [4] existing in the case of an interface between air and a uniform metal). Such surface plasmons were observed through the analysis by Schwartz and Schaich [8], were described by Dobson and Harris [9] and further elaborated upon by Nazarov [10] and Nazarov and Nishigaki [11]. Possibly surprisingly no detailed theoretical examination of the spectrum has been done subsequent to the above papers.

We briefly review the foundations of the model.

(i) The electrons are considered as a continuous fluid characterized by a pressure P and a temperature T obeying the equations of an ideal, near-degenerate Fermi gas.

(ii) Conservation of mass familiar from fluid mechanics holds in its usual form. If $\mathbf{v}$ is the fluid velocity and $\tilde{\rho}$ the mass density

$$\frac{\partial \tilde{\rho}}{\partial t} + \nabla \cdot (\tilde{\rho}\mathbf{v}) = 0. \tag{1}$$

(iii) Gauss' law of electrostatics reads

$$\nabla \cdot \mathbf{E} = 4\pi e(n - N_i), \tag{2}$$

where N_i is the number density of the fixed ionic background.

(iv) Newton's law for the electronic fluid reads

$$\tilde{\rho}\frac{\partial \mathbf{v}}{\partial t} + \tilde{\rho}\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} = en\mathbf{E} - \nabla P. \tag{3}$$

(v) Overall charge neutrality holds. In other words the motion of the electron fluid takes place against an immobile background of neutralizing charge.

(vi) The electron fluid obeys the equations of a low-temperature ideal Fermi gas. In particular the speed of sound and electron gas pressure are given by

$$\beta^2 = \frac{v_F^2}{3}\left(1 + \frac{5\pi^2}{12}\frac{T^2}{T_F^2} + \cdots\right), \tag{4}$$

$$P = \frac{2}{5}n\varepsilon_F\left(1 + \frac{5\pi^2}{12}\frac{T^2}{T_F^2} + \cdots\right) \tag{5}$$

where ε_F, v_F, T_F are the Fermi energy, Fermi velocity, and Fermi temperature respectively and n is the electron number density.

Further we add a (vii) which reflects our purposes rather than the limitations of the model: Retardation is to be neglected or in other words $c \to \infty$ from the outset. The electric field is longitudinal and hence derivable from a potential:

$$\mathbf{E} = -\nabla\Phi. \tag{6}$$

Items (i)–(vii) allow for quite rich physics. The electron fluid in its equilibrium state is characterized by an electron number density n_{eq}. In the works cited n_{eq} is taken to be a constant function of position (except of course for θ-function discontinuities in the presence, for example, of an air–metal interface). In the present work we treat a plasma slab with a smoothly varying equilibrium electron density $n_{eq}(z)$. This seemingly innocuous generalization gives rise to severe mathematical complications when we come to examine deviations from equilibrium. Here the form of $n_{eq}(z)$ is in principle arbitrary except that it tends to a constant deep inside the metal.

In Section 2 we use the equilibrium properties of the metal, i.e. the balance between pressure and electrostatic forces, to calculate the fixed background ionic charge density $N_i(z)$ appropriate for a given $n_{eq}(z)$. The non-uniform electron number density gives rise (at equilibrium) to a pressure gradient that must be countered by an electric field that has a non-vanishing equilibrium value; the latter is given by Eq. (11). Charge neutrality then requires two spikes of immobile charge situated at the interfaces. In actual experimental practice the reverse procedure would take place: one would