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Quantum field as a quantum cellular automaton: The Dirac free evolution in one dimension

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HIGHLIGHTS

- The free Dirac field in one space dimension as a quantum cellular automaton.
- Large scale limit of the automaton and the emergence of the Dirac equation.
- Dispersive differential equation for the evolution of smooth states on the automaton.
- Optimal discrimination between the automaton evolution and the Dirac equation.

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ABSTRACT

We present a quantum cellular automaton model in one space-dimension which has the Dirac equation as emergent. This model, a discrete-time and causal unitary evolution of a lattice of quantum systems, is derived from the assumptions of homogeneity, parity and time-reversal invariance.

The comparison between the automaton and the Dirac evolutions is rigorously set as a discrimination problem between unitary channels. We derive an exact lower bound for the probability of error in the discrimination as an explicit function of the mass, the number and the momentum of the particles, and the duration of the evolution. Computing this bound with experimentally achievable values, we see that in that regime the QCA model cannot be discriminated from the usual Dirac evolution.

Finally, we show that the evolution of one-particle states with narrow-band in momentum can be efficiently simulated by a dispersive differential equation for any regime. This analysis allows for a comparison with the dynamics of wave-packets as it is described by the usual Dirac equation.

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This paper is a first step in exploring the idea that quantum field theory could be grounded on a more fundamental quantum cellular automaton model and that physical dynamics could emerge from quantum information processing. In this framework, the discretization is a central ingredient and not only a tool for performing non-perturbative calculation as in lattice gauge theory. The automaton model, endowed with a precise notion of local observables and a full probabilistic interpretation, could lead to a coherent unification of a hypothetical discrete Planck scale with the usual Fermi scale of high-energy physics.

1. Introduction

The major problem of developing a quantum theory of gravity, whose effects should become relevant at the Planck scale, seems to require a deep reconsideration of the spacetime structure. Recently alternative models of spacetime are gathering increasing attention. We can cite for example the loop quantum gravity model by Rovelli, Smolin and Ashtekar [1–3], the causal sets approach of Bombelli et al. [4], the noncommutative spacetime of Connes [5], the quantized spacetime of Snyder [6], the doubly-special relativity of Camelia in [7,8] along with the deformed special relativity models of Smolin and Magueijo in [9]. Some of these approaches are even considered for experimental tests, see for example the recent experiment proposals by Hogan [10,11] and Brukner [12]. Moreover, the finiteness of the entropy of a black hole [13,14], which implies that the number of bits of information that can be stored is finite, has led to the idea that space–time at the Planck scale could be discrete and that the amount of information in a finite volume must always be finite.

In this work, following the ideas proposed in Refs. [15–19], we assume that at the Planck scale physical dynamics occurs on a discrete lattice and in discrete time steps. Considering for simplicity the one-dimensional case, the lattice is a chain of sites equally spaced with a period assumed to be equal to the Planck length ℓ_P, while a single time step is equivalent to a Planck time τ_P. Each site x corresponds to a quantum system whose dynamics is described by a *quantum cellular automaton* (QCA). The QCA generalizes the notion of cellular automaton of von Neumann [20] to the quantum case, with cells of quantum systems interacting with a finite number of nearest neighboring cells via a unitary operator describing the single step evolution.

One of the first theoretical notion of QCA appeared in Ref. [21], and later in [22,23] where it was referred to as linear quantum cellular automata, while the notion of QCA as a mean for simulating quantum physical systems originally appeared in Refs. [24–26]. Since then the QCAs have been a quantum-computer-science object of investigation with a rigorous formulation and relevant results about their general structure [27–29]. Moreover, in the field of quantum information, particular attention is devoted to the so-called *quantum walks* (QWs) which describe the quantum evolution of one particle moving on a discrete lattice and which correspond the one particle sector of QCAs with linear evolution [30–32].[1] This interest is motivated by the use of QWs in the design of quantum algorithms: in Ref. [33] Childs et al. proved that QWs provide an exponential speedup for an oracular problem and QWs are also known to provide polynomial speedups for many relevant problems [34–36].

The idea of modeling the physical evolution at the Planck scale on a discrete background first appeared in the work of 't Hooft [37]. However, in his work the automaton is classical, and it describes a deterministic discrete theory underlying quantum theory. Then the idea of using QWs for the simulation of Lorentz-covariant differential equations appeared in the pioneering works of Succi and

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[1] Notice that in Ref. [30] the word quantum cellular automaton appears for the first time. However, the model presented in the paper describe the one-particle evolution and is technically a QW.