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Gauge natural formulation of conformal gravity

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ABSTRACT

We consider conformal gravity as a gauge natural theory. We study its conservation laws and superpotentials. We also consider the Mannheim and Kazanas spherically symmetric vacuum solution and discuss conserved quantities associated to conformal and diffeomorphism symmetries.

1. Introduction

General Relativity is a self-consistent covariant theory for gravity which is able to successfully describe gravity at many scales. Its predictions agree with observations at the Solar System and astrophysical scales. However, at galactic and cosmological scales, one needs to introduce a huge amount of dark sources in the form of dark matter and dark energy in order to model phenomenological aspects such as galaxy rotation curves or properly describe structures formation. On the other hand what dark sources are at fundamental level remains a mystery. While looking for fundamental models for dark sources is an option, one can consider desirable alternative theories of gravity which account for dark sources effectively as pure gravitational effects.

Philip Mannheim proposed a conformally invariant theory of gravity based on the conformal Weyl tensor [1]. This theory is worth being considered among the class of modified or extended gravitational models. Besides being a candidate for physical modeling of gravitational phenomena at different scales it also provides a test for a more general understanding of the relation between covariant theories and observations. We shall show that conformal gravity can be included in the more general framework of gauge natural theories adding conformal invariance right into the game at kinematical level. In this

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way the theory is formally very similar to a gauge theory and the geometrical meaning of its fields is rendered explicitly and suitably encoded into a principal bundle over the spacetime manifold; see [2] for a more detailed discussion about the physical meaning of these conformal transformations in a similar though different kinds of theories.

Depending on the context, two related but distinct physical transformations are denoted as *conformal* in the literature. While conformal transformations are often considered (as in conformal field theories) as particular coordinate transformations which leave the metric structure unchanged modulo a (positive) factor, they are sometimes also introduced (especially in conformal gravity) as transformations which do not affect the spacetime point while they change the metric field by a pointwise (positive) factor. According to the first viewpoint conformal transformations are a special class of diffeomorphisms. In the second viewpoint conformal transformations are gauge transformations acting on fields alone (i.e. they are vertical transformations on the configuration bundle).

The differences between these two viewpoints may be considered trivial though they have a certain amount of consequences which are worth addressing. A trivial difference is that conformal transformations on spacetime (i.e. the first viewpoint) form a group which is bigger than isometries and smaller than diffeomorphisms. While it can be meaningful to try and extend a special relativistic theory to be covariant which respect to the bigger group of these conformal transformations (as essentially one does in conformal field theories) it makes relatively no sense to consider a conformal version of a generally covariant theory (as one does in conformal gravity). A generally covariant theory is already covariant with respect to all diffeomorphisms (including the subgroup of conformal spacetime transformations). In other words, in conformal gravity the only allowed viewpoint is to regard conformal transformations as gauge transformations. This of course produces ambiguous notations which need to be dealt with care. It is our opinion that the gauge natural approach provides a good foundation of both viewpoints and it allows to avoid notational ambiguities. However, hereafter we shall not discuss this issue in detail. We shall only present the gauge natural formulation of conformal gravity and discuss conservation laws, leaving the foundational issues for a future more general investigation.

As we shall see, Weyl tensor comes in quite naturally as a consequence of symmetry requirements on dynamics and a canonical treatment of conservation laws is a free token from gauge natural framework (see [3–5]). Conserved currents for gauge natural theories are exact differential forms (on-shell), which do admit a superpotential. Thus, it can be developed as a canonical way of finding conserved quantities. Therefore, after having set up a well-founded geometrical framework, one has a useful tool for analyzing physical phenomena such as the gravitational lensing.

Especially in conformal gravity the issue of conserved quantities would be particularly important to be understood generally for applications for example to gravitational lensing (and this paper is meant to be in preparation for such an analysis). In fact solutions in conformal gravity are particularly poorly understood. First of all being the theory conformally invariant any weak field approximation will still be conformally invariant. For example one obtains a good theory of motion of test light rays (which is in fact conformally invariant) while the Newtonian limit of test particles would depend on the details of some gauge fixing. Since the masses as defined in astrophysics are definitely not conformally invariant (the third Kepler law is definitely not conformally invariant) these notions cannot simply be obtained by Newtonian limit (or at least not *only* in terms of Newtonian limit but they would depend on the details of some gauge fixing). The same attention needs to be paid if one defines masses in terms of conservation laws (as we shall see the superpotentials are in fact conformally invariant as their integrals will be).

On the other hand in solutions which are not asymptotically flat (as they are in conformal gravity but also in many cases of GR models) a great effort has to be spent in order to define deflection angles so that they make geometrical sense. One way would be to compute the angle between received light rays with and without the gravitational lens. This procedure in particular needs a complete control on what it means to *switch off the lens*, i.e. which parameters appearing in the solution are related to the localized source and which are related to the asymptotic behavior. This complete control is still to be obtained, in conformal gravity as in other models, and should be based on a careful analysis of conservation laws which is what we start to do hereafter.