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Entanglement and the process of measuring the position of a quantum particle

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HIGHLIGHTS

- We explore entanglement features of a quantum position measurement.
- We consider instantaneous and finite-duration measurements.
- We evaluate the entanglement of exact time-dependent particle–pointer states.

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ABSTRACT

We explore the entanglement-related features exhibited by the dynamics of a composite quantum system consisting of a particle and an apparatus (here referred to as the "pointer") that measures the position of the particle. We consider measurements of finite duration, and also the limit case of instantaneous measurements. We investigate the time evolution of the quantum entanglement between the particle and the pointer, with special emphasis on the final entanglement associated with the limit case of an impulsive interaction. We consider entanglement indicators based on the expectation values of an appropriate family of observables, and also an entanglement measure computed on particular exact analytical solutions of the particle–pointer Schrödinger equation. The general behavior exhibited by the entanglement indicators is consistent with that shown by the entanglement measure evaluated on particular analytical solutions of the Schrödinger equation. In the limit of instantaneous measurements the system's entanglement dynamics corresponds to that of an ideal quantum measurement process. On the contrary, we show that the entanglement evolu-

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tion corresponding to measurements of finite duration departs in important ways from the behavior associated with ideal measurements. In particular, highly localized initial states of the particle lead to highly entangled final states of the particle–pointer system. This indicates that the above mentioned initial states, in spite of having an arbitrarily small position uncertainty, are not left unchanged by a finite-duration position measurement process.

0. Introduction

Quantum entanglement [1,2] and the quantum measurement process [3,4] are two closely related and fundamentally non-classical aspects of quantum physics. If initially the system being measured is described by a pure state (which, consequently, is factorized from the initial, standard state of the measuring apparatus), the measurement process in general generates entanglement between the system and the apparatus [5,6]. Therefore, after the measurement takes place (but before the result of the measurement is "read") the system and the apparatus are, in general, in an entangled state (except in the case of an ideal measurement with the system starting in an eigenstate of the observable being measured). Within the standard quantum formalism one can consider the measurement of general physical observables described by appropriate hermitian operators acting on the relevant Hilbert space. However, it is generally acknowledged that the measurement of the position of quantum particles plays a particularly fundamental role among the set of all possible physical measurements. In fact, most, if not all, physical measurements can be reduced to the measurement of the position of some particle (for instance, a pointer in the measuring apparatus) [7,8]. This is one of the main reasons why position observables play a central role in many approaches to the quantum measurement problem and related aspects of the foundations of quantum mechanics. Among the interesting position-centered contributions to these fundamental issues we can mention the de Broglie–Bohm pilot wave approach to quantum mechanics [9–11], the Ghirardi–Rimini–Weber model of wave-function collapse [12], non-linear modifications of Schrödinger equation describing the continuous measurement of a particle's position [13,14], the Fisher information-based derivation of the fundamental Lagrangians leading to relativistic wave equations [15], and the entropic-dynamics approach to quantum evolution [16].

A central point concerning the quantum measurement problem is whether one regards the measurement process as arising from a physical interaction between the system and the measuring apparatus describable by the standard, linear Schrödinger equation. A useful tool for analyzing the conceptual issues associated with this point of view is given by the celebrated von Neumann model for quantum measurements [17–19]. In this model the measuring apparatus is characterized by one, single relevant coordinate Q (the "pointer" coordinate). If the system being measured is described by a coordinate R, then the von Neumann model assumes that the system–apparatus Hamiltonian has an interaction term of the form,

$$G\delta(t)F\left(R, \frac{\hbar}{i}\frac{\partial}{\partial R}\right)\frac{\hbar}{i}\frac{\partial}{\partial Q}, \tag{1}$$

where $F\left(R, \frac{\hbar}{i}\frac{\delta}{\delta R}\right)$ corresponds to the observable being measured (which is here expressed as a function of the position and the momentum observables) and G is a coupling constant. Notice that this interaction term is time-dependent, and describes an impulsive interaction that is switched on at the instant $t = 0$. This means that the interaction between the system and the apparatus has a very short duration and is very strong. Therefore, under the impulsive assumption, the contribution to the evolution of the system–apparatus composite due to the "free" Hamiltonians associated with the two parts (i.e., the system and the apparatus) can be neglected during the measurement process.