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A new functional flow equation for Einstein–Cartan quantum gravity

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ABSTRACT

We construct a special-purpose functional flow equation which facilitates non-perturbative renormalization group (RG) studies on theory spaces involving a large number of independent field components that are prohibitively complicated using standard methods. Its main motivation are quantum gravity theories in which the gravitational degrees of freedom are carried by a complex system of tensor fields, a prime example being Einstein–Cartan theory, possibly coupled to matter. We describe a sequence of approximation steps leading from the functional RG equation of the Effective Average Action to the new flow equation which, as a consequence, is no longer fully exact on the untruncated theory space. However, it is by far more "user friendly" when it comes to projecting the abstract equation on a concrete (truncated) theory space and computing explicit beta-functions. The necessary amount of (tensor) algebra reduces drastically, and the usually very hard problem of diagonalizing the pertinent Hessian operator is sidestepped completely. In this paper we demonstrate the reliability of the simplified equation by applying it to a truncation of the Einstein–Cartan theory space. It is parametrized by a scale dependent Holst action, depending on a O(4) spin-connection and the tetrad as the independent field variables. We compute the resulting RG flow, focusing in particular on the running of the Immirzi parameter, and compare it to the results of an earlier computation where the exact equation had been applied to the same truncation. We find consistency between the two approaches and provide further evidence for the conjectured non-perturbative renormalizability (asymptotic safety) of quantum Einstein–Cartan gravity. We also investigate a duality symmetry relating small and large values of the Immirzi parameter ($\gamma \to 1/\gamma$)

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which is displayed by the beta-functions in the absence of a cosmological constant.

1. Introduction

Searching for a fundamental quantum theory of gravity is a long-standing and on-going quest of modern physics. Since the perturbative non-renormalizability of a quantum field theory of metric gravity was shown [1–3] various different approaches as e.g. string theory, loop quantum gravity (LQG) or Asymptotic Safety have been pursued in order to find a solution to this problem. These approaches differ in which of the constituents are accounted for the failure of the perturbative renormalizability of metric gravity, i.e. quantum field theory, perturbation theory or the choice of variables that serve as a carrier of the fundamental gravitational degrees of freedom. While string theory leaves the framework of quantum field theory by choosing strings of finite length as fundamental degrees of freedoms, it essentially sticks to perturbative methods. LQG, in contrast, is constructed non-perturbatively relying on a Hamiltonian formalism that uses a special choice of variables to parametrize phase space, which are quantized canonically.

The Asymptotic Safety scenario for gravity is the most conservative approach among these as it attributes the theory's perturbative non-renormalizability only to the unjustified use of perturbation theory. It can be seen as a specific, particularly natural ultraviolet (UV) completion of the effective field theory framework pioneered by Donoghue [4,5]. As conjectured by Weinberg [6] metric gravity in four dimensions could be well-defined in the UV at a non-Gaussian fixed point (NGFP) in the space of all actions, i.e. a fixed point of the renormalization group flow, that corresponds to an interacting theory. If this conjectured NGFP exists but is located at large couplings outside the realm of perturbation theory it cannot be found or examined by perturbative methods, even though the description of gravity as a conventional quantum field theory remains valid at all scales.

Thus, by definition, the investigation of the Asymptotic Safety scenario in gravity can only be carried out using non-perturbative methods in quantum field theory. An appropriate tool for this investigation is the functional renormalization group equation (FRGE) for the effective average action (EAA) which was first derived for scalar [7] and Yang–Mills theory [8–11], and later on for gravity [12]. This exact functional equation for a running effective action functional typically cannot be solved in full generality but allows for non-perturbative approximations by choosing an ansatz for the form of the action functional, a so-called truncation. The FRGE rendered possible an investigation of the renormalization group (RG) flow of metric gravity in arbitrary dimensions of spacetime. Since then numerous studies of different approximations have been carried out,[1] all of which indicate the existence of a NGFP for metric gravity in $d = 4$ spacetime dimensions. Also the inclusion of matter fields, that causes divergences in the perturbative approach already at the one-loop level, has been explored in some detail [17–20], and it was found that Asymptotic Safety of gravity is compatible with the matter content of the Standard Model, although general bounds on the number of matter fields were found to exist. Taken together an impressive amount of evidence for Asymptotic Safety of metric gravity at a NGFP in the space of diffeomorphism invariant action functionals has been collected, such that it is likely to be a true feature of this theory space and not merely an artifact of the approximations applied. Theories of metric gravity whose continuum limit is taken at this NGFP are called Quantum Einstein Gravity (QEG).

Despite its success in metric gravity, it is important to keep in mind that the concept of Asymptotic Safety is neither linked nor restricted to using the metric as the carrier field for the gravitational degrees of freedom. In the search for an asymptotically safe theory of gravity the only restriction to

[1] For an overview of this field of research we refer to the review articles [13–16].