Annals of Physics 350 (2014) 105–111

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Landau quantization in the spinning cosmic string spacetime

C.R. Muniz [a,*], V.B. Bezerra [b], M.S. Cunha [c]

[a] Grupo de Física Teórica (GFT), Universidade Estadual do Ceará, UECE-FECLI, Iguatu-CE, Brazil
[b] Departamento de Física, Universidade Federal da Paraíba, Caixa Postal 5008, CEP 58051-970, João Pessoa-PB, Brazil
[c] Grupo de Física Teórica (GFT), Universidade Estadual do Ceará-UECE, CEP 60740-000, Fortaleza-CE, Brazil

HIGHLIGHTS

- Solution of the wave equation characterizing the problem.
- Energy levels of the particle in spacetime of the structureless string.
- Expression for an analogous of the quadratic Zeeman effect.
- Energy levels of the particle in spacetime of the string with internal structure.
- Evidence of the string structure by the internal existence of the vacuum energy.

ARTICLE INFO

Article history:
Received 31 March 2014
Accepted 8 July 2014
Available online 14 July 2014

Keywords:
Landau level
Spacetime rotation
Cosmic string

ABSTRACT

We analyze the quantum phenomenon arising from the interaction of a spinless charged particle with a rotating cosmic string, under the action of a static and uniform magnetic field parallel to the string. We calculate the energy levels of the particle in the non-relativistic approach, showing how these energies depend on the parameters involved in the problem. In order to do this, we solve the time independent Schrödinger equation in the geometry of the spinning cosmic string, taking into account that the coupling between the rotation of the spacetime and the angular momentum of the particle is very weak, such that makes sense to apply the Schrödinger equation in a curved background whose metric has an off diagonal term which involves time and space. It is also

* Corresponding author.
E-mail addresses: celiomuniz@yahoo.com, celiomuniz131@gmail.com (C.R. Muniz).

http://dx.doi.org/10.1016/j.aop.2014.07.017

assumed that the particle orbits sufficiently far from the boundary of the region of closed timelike curves which exist around this topological defect. Finally, we find the Landau levels of the particle in the presence of a spinning cosmic string endowed with internal structure, *i.e.*, having a finite width and uniformly filled with both material and vacuum energies.

1. Introduction

Spinning cosmic strings are stable one-dimensional topological defects, infinitely long and straight, characterized by an angular parameter α that depends on their linear mass density μ and by a linear density of angular momentum **J**. These structures are the stationary counterpart of the static cosmic strings, in which $\alpha = 1 - 4G\mu$, probably arising during the very early stages of the known universe, when the first decoupling between the fundamental interactions described by the Standard Model of Elementary Particles occurred [1].

Such objects were studied for the first time as vacuum solutions of General Relativity through the Kerr spacetime reduced to $1 + 2$

dimensions [2]. Shortly after, these three-dimensional solutions were naturally extended to the 4-dimensional spacetime [3] in order to describe spinning cosmic strings. Like the static ones, their geometry is locally flat, but not globally. There is a singularity which coincides with the localization of the string, along which the Riemann curvature is infinity.

The spacetime of spinning cosmic strings has peculiar and nontrivial topological properties, which arise from its conical geometry, as well as from the rotation of the spacetime. Like in the case of their simple [4,5] or more involved [6] static analogues, these properties result in some interesting phenomena, as gravitomagnetism and the gravitational Aharonov–Bohm effect, both arising at a purely classical level (*i.e.*, non-quantum) [7].

Spinning cosmic strings can also present the internal structure [8] and are surrounded by an exotic region that allows the formation of closed timelike curves (CTCs) [9], which are problematic from the point of view of the violation of causality. The boundary that defines this region is at a distance proportional to J/α measured from the cosmic string and provides a natural boundary condition for the involved fields. The spinning cosmic string was also studied in Cartan–Einstein's theory [10,11] and teleparallel gravity [12], in which the regions of CTCs were also examined. There are also studies on these objects in the context of extra dimensions, including its causal structure and raising criticisms about the possibility of the existence of the region of CTCs [13].

About the Landau levels of particles localized in the spacetime of spinning cosmic string, there is little literature [14], as opposed to what happens with static cosmic strings [15–18], despite of the occurrence of analogues to the spinning cosmic string in condensed matter physics, more specifically vortices in superfluids [19,20]. We think that the present paper gives a contribution in the direction to suppress this gap, helping to elucidate the theoretical Landau quantization of charged particles placed in the spacetime of spinning cosmic strings, including the possibility of these having an internal structure. The approach consists in the resolution of the Schrödinger equation in the appropriate metric, considering an approximation in which the particle is far from the boundary of the CTCs region. The levels of energy are extracted from the found solutions. Finally, we will analyze the problem of the Landau levels in the spacetime of a spinning cosmic string with an internal structure, whose interior region is uniformly filled with both matter and vacuum energies, the latter represented by a cosmological constant.

This paper is organized as follows: Section 2 details the methodology, Section 3 presents the results and discusses them and Section 4 closes the paper with the conclusions.