Annals of Physics 350 (2014) 179–197

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Superconducting cosmic strings and one dimensional extended supersymmetric algebras

V.K. Oikonomou *

Department of Mechanical Engineering, Technological Education Institute of Serres, 62124 Serres, Greece
Department of Theoretical Physics, Aristotle University of Thessaloniki, 54124 Thessaloniki, Greece

ARTICLE INFO

Article history:
Received 21 February 2014
Accepted 18 July 2014
Available online 23 July 2014

Keywords:
Supersymmetric quantum mechanics
Localized zero modes
Low dimensional supersymmetry
Quantum mechanics

ABSTRACT

In this article we study in detail the supersymmetric structures that underlie the system of fermionic zero modes around a superconducting cosmic string. Particularly, we extend the analysis existing in the literature on the one dimensional $N = 2$ supersymmetry and we find multiple $N = 2$, $d = 1$ supersymmetries. In addition, compact perturbations of the Witten index of the system are performed and we find to which physical situations these perturbations correspond. More importantly, we demonstrate that there exists a much more rich supersymmetric structure underlying the system of fermions with N_f flavors and these are N-extended supersymmetric structures with non-trivial topological charges, with "N" depending on the fermion flavors.

0. Introduction and motivation

Domain walls, cosmic strings and monopoles, are theoretical predictions of various grand unified models beyond the Standard Model [1,2]. Among the three aforementioned topological defects, the most cosmologically acceptable are cosmic strings with a high energy symmetry breaking scale. On the other hand, monopoles and domain walls usually lead to cosmological inconsistencies when observable outcomes, that these defects impose, are taken into account. However, topologically unstable domain walls are phenomenologically acceptable for various reasons. Furthermore, this kind of domain walls are particularly plausible since the presence of a low tension domain wall network in the

* Correspondence to: Department of Mechanical Engineering, Technological Education Institute of Serres, 62124 Serres, Greece.

E-mail address: voiko@physics.auth.gr.

http://dx.doi.org/10.1016/j.aop.2014.07.020
0003-4916/

universe can provide a somehow natural and non-exotic alternative to existing dark energy models related to modified gravity theories (for an important stream of papers on modified gravity theories see [3] and references therein).

A simplified version of the four dimensional super-Poincare algebra is supersymmetric quantum mechanics (abbreviated to SUSY QM hereafter). SUSY QM has become an independent research field, with numerous applications in various research areas (for informative reviews see [4]). Mathematical aspects of Hilbert spaces corresponding to SUSY QM systems and in addition applications to various quantum mechanical systems were studied in [5–8]. Moreover, extended supersymmetries and their relation to harmonic superspaces or gravity, were presented in [9,10]. Applications of SUSY QM to scattering appear in [11] and specific features of supersymmetry breaking were studied in [12]. The connection between central charge extended SUSY QM and global four dimensional spacetime supersymmetry were studied in [10].

SUSY QM was originally introduced to model supersymmetry breaking in a four dimensional quantum field theory framework [13]. Supersymmetry is a very successful theoretical tool in the beyond the Standard Model physics and also in string theory. The theoretical and phenomenological implications of supersymmetry are so important that rendered it an important ingredient of various research fields [14–22]. Up to date there is no experimental verification of supersymmetry and therefore supersymmetry must be broken in our four dimensional world. With regards to supersymmetry breaking, there exist various elegant ways to break supersymmetry and the interested reader may consult [14,15] and references therein.

The presence of transverse fermionic zero modes around a cosmic string render the string superconducting [23]. Superconducting strings play an important role in cosmological frameworks that originate from a grand unified theory model. As is known [23], superconducting current loops coming from GUT superconducting strings at an early epoch, may explain the big voids in the distribution of galaxies and also the formation of galaxies themselves. The connection of the fermionic zero modes around a superconducting cosmic string to $N = 2, d = 1$ supersymmetry was pointed out in Ref. [24]. As was shown there, by calculating the Witten index of the underlying supersymmetric fermionic system, suffices to decide whether the string is superconducting or not.

The purpose of this article is to further elaborate on the supersymmetric structures that underlie the fermionic system around superconducting cosmic strings. Particularly, we shall demonstrate that, when we have N_f flavors of fermions, we actually have a total number of $N_f!/(N_f - 2)! + N_f$ different $N = 2$ SUSY QM algebras underlying the system. We shall see that this gives the Hilbert space of the SUSY QM system an additional symmetry, which actually is a product of global $U(1)$ symmetries. Using the $N = 2, d = 1$ SUSY QM algebra we may form higher reducible representations of these algebras. A trivial extension of each $N = 2$ SUSY QM algebra by introducing a trivial central charge is performed too. In addition, we shall study how the Witten index behaves under compact and odd perturbations. This can only be done in the case a background static magnetic field is turned on, with the only non-zero component being $\mathcal{B}_z$. In addition, we provide a criterion to answer if a cosmic string is superconducting, in the case the index I_q, frequently used in the bibliography [23,24], is equal to zero. Finally, we shall demonstrate that the $N = 2$ supersymmetries are not the only supersymmetric structure underlying the fermionic system. Particularly, depending on the fermion flavors, we shall see that there exist extended supersymmetric structures with non-trivial topological charge. These can be $N = 4, 6, \ldots$ depending on the total number of fermion flavors. With topological charge we mean an operator that is equal to the anticommutator of Fermi charges (supercharges), following the terminology of Refs. [18,17,19]. As we shall see, these topological charges can be central charges of the supersymmetry algebra when some conditions are satisfied.

This paper is organized as follows: In Section 1, we briefly review the theoretical framework of fermions around superconducting strings and also the results from previous work on the relation between the fermionic system with $N = 2, d = 1$ SUSY QM. In Section 2, we generalize the results of Section 1 in the case we have N_f flavors of fermions and we find multiple $N = 2, d = 1$ supersymmetries in the system. In addition, we demonstrate that the fermionic quantum system has a global R-symmetry which is a product of global $U(1)$ symmetries. Moreover, we perform compact odd perturbations on the Witten index and we also show that we can form higher representations of $N = 2$, $d = 1$ supersymmetric algebra, by combining the multiple $N = 2$ we found previously. Furthermore,