Annals of Physics 350 (2014) 211–224

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Recovering entanglement by local operations

A. D'Arrigo [a,b,c,*], R. Lo Franco [d,c], G. Benenti [e,f], E. Paladino [b,a,c,g], G. Falci [b,a,c,g]

[a] CNR-IMM UOS Università (MATIS), Consiglio Nazionale delle Ricerche, Via Santa Sofia 64, 95123 Catania, Italy

[b] Dipartimento di Fisica e Astronomia, Università degli Studi Catania, Via Santa Sofia 64, 95123 Catania, Italy

[c] Centro Siciliano di Fisica Nucleare e Struttura della Materia (CSFNSM), Via Santa Sofia 64, 95123 Catania, Italy

[d] Dipartimento di Fisica e Chimica, Università di Palermo, via Archirafi 36, 90123 Palermo, Italy

[e] CNISM and Center for Nonlinear and Complex Systems, Università degli Studi dell'Insubria, Via Valleggio 11, 22100 Como, Italy

[f] Istituto Nazionale di Fisica Nucleare, Sezione di Milano, via Celoria 16, 20133 Milano, Italy

[g] Istituto Nazionale di Fisica Nucleare, Sezione di Catania, Viale S. Sofia 64, 95123 Catania, Italy

A R T I C L E   I N F O

Article history:
Received 1 February 2014
Accepted 18 July 2014
Available online 23 July 2014

Keywords:
Quantum information
Entanglement
Decoherence
Open quantum systems

A B S T R A C T

We investigate the phenomenon of bipartite entanglement revivals under purely local operations in systems subject to local and independent classical noise sources. We explain this apparent paradox in the physical ensemble description of the system state by introducing the concept of "hidden" entanglement, which indicates the amount of entanglement that cannot be exploited due to the lack of classical information on the system. For this reason this part of entanglement can be recovered without the action of *non-local* operations or *back-transfer* process. For two noninteracting qubits under a low-frequency stochastic noise, we show that entanglement can be recovered by local pulses only. We also discuss how hidden entanglement may provide new insights about entanglement revivals in non-Markovian dynamics.

* Corresponding author at: CNR-IMM UOS Università (MATIS), Consiglio Nazionale delle Ricerche, Via Santa Sofia 64, 95123 Catania, Italy.

E-mail addresses: antonio.darrigo@dmfci.unict.it, ant.darrigo@gmail.com (A. D'Arrigo).

http://dx.doi.org/10.1016/j.aop.2014.07.021
0003-4916/

1. Introduction

Entanglement, arguably the most peculiar feature of quantum mechanics, plays a key role in several quantum information and communication applications, including teleportation, quantum dense coding, private key distribution, and reduction of communication complexity [1–4]. To work properly, all the above tasks generally require pure maximally entangled states. Since entanglement cannot be generated by Local Operations and Classical Communication (LOCC), entangled states must be generated somewhere, and then they have to be distributed among different parties, possibly far away from each other (*transmission*) [3,4]. Once entanglement has been distributed, it can be used immediately or stored for later use (*storage*). Systems physically supporting entangled states, unavoidably interact with the environment, both during transmission and storage, and therefore undergo noisy processes that deteriorate entanglement. Quantification of entanglement losses is thereby necessary for all practical purposes.

For a pure state $\rho = |\psi\rangle\langle\psi|$, bipartite entanglement between subsystems A and B is unambiguously defined as the *entropy of entanglement* $E(|\psi\rangle\langle\psi|) = S(\rho_A) = S(\rho_B)$, where $S(\rho_i)$ is the von Neumann entropy of one of the two reduced states, $\rho_A = \mathrm{Tr}_B\rho$ and $\rho_B = \mathrm{Tr}_A\rho$. The quantification of entanglement for mixed states is a much more complicated and still open problem [3,4]. The difficulty roots in the fact that a mixed state ρ may be decomposed into an ensemble of pure states $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$, with $p_i > 0$ and $\sum_i p_i = 1$, in infinite different ways. The arbitrariness of the decomposition renders any quantification of mixed-state entanglement cumbersome, since it requires an optimization over all possible decompositions.

In this article, we address the issue of the occurrence of entanglement revivals of a bipartite system, initially prepared in an entangled state, when the two subsystems are noninteracting and affected by local independent classical noise sources and local operations (see Fig. 1(a)). In the absence of non-local operations, entanglement cannot be generated neither back-transferred to the system from the classical environment. Nevertheless, during the system dynamics, entanglement quantified by some measure E may start to increase at some time $\bar{t}$ [5,6] as illustrated in Fig. 1(b). As we will explain, the increase of entanglement must be attributed to the manifestation of pre-existing quantum correlations, that were already present before $\bar{t}$. The density operator formalism does not capture the presence of these quantum correlations, thus they are in some sense *hidden*. Here we point out that the existence of these correlations is enlightened if the system is described as a physical ensemble of states and we introduce the concept of *hidden entanglement*.

This paper is structured as follows. In Section 2 we introduce a definition of hidden entanglement (HE) and illustrate the usefulness of this concept by a simple example. In Section 3 we show that HE between two noninteracting qubits subject to a non-Markovian stochastic process can be recovered by *local* pulses (acting only on one qubit). The nature of the observed entanglement revivals and the relation of this phenomenon with the environment being classical or quantum, are clarified. In Section 4 we critically discuss some key points related to the definition of HE. In particular, we show that entanglement recovery does not violate the monotonicity axiom: *entanglement cannot increase under LOCC* [3,4,7]. We draw our conclusions in Section 5.

2. Hidden entanglement

Let us consider a bipartite system described by an ensemble of states $\mathcal{A} = \{(p_i, |\psi_i\rangle)\}$. That is, we know the statistical distribution of the bipartite pure states $\{|\psi_i\rangle\}$, occurring with probabilities $\{p_i\}$, so that $\rho = \sum_i p_i|\psi_i\rangle\langle\psi_i|$, but the state of any individual system in the ensemble is unknown. The average entanglement of $\mathcal{A}$ is defined as [7–10]:

$$E_{\mathrm{av}}(\mathcal{A}) = \sum_i p_i E(|\psi_i\rangle\langle\psi_i|). \tag{1}$$

If each system in the ensemble evolves during time t under LOCC, the maximum amount of entanglement of the corresponding density operator $\rho(t)$ can never overcome the initial value $E_{\mathrm{av}}(\mathcal{A})$. This statement can be proved by the following simple argument. Suppose Charlie prepares a bipartite