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Classical states of an electric dipole in an external magnetic field: Complete solution for the center of mass and trapped states

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HIGHLIGHTS

- Bound states without turning points.
- Lagrangian Formulation for an electric dipole in a magnetic field.
- Motion of the center of mass and trapped states.
- Constants of motion: pseudomomentum and energy.

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ABSTRACT

We study the classical behavior of an electric dipole in the presence of a uniform magnetic field. Using the Lagrangian formulation, we obtain the equations of motion, whose solutions are represented in terms of Jacobi functions. We also identify two constants of motion, namely, the energy $\mathcal{E}$ and a *pseudomomentum* $\vec{C}$. We obtain a relation between the constants that allows us to suggest the existence of a type of bound states without turning points, which are called *trapped states*. These results are consistent with and complementary to previous results.

1. Introduction

In the present day, many specialists study the world at the molecular scale. Nanotechnology is slowly exploring molecular rotors, and applications of this concept are extensive. Using electric fields,

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molecules can change in orientation and/or remain controlled [1–3]. Molecular-level devices can be obtained from the conversion of energy into controlled motion; nevertheless, it is difficult to repeat this process using a mechanical molecular motor, although it is common in biological systems. For the time being, it is expected that the physical principles at the scale of a molecular engine can be identified by applying rotor dynamics in two dimensions. These rotors are modeled as electric dipoles in electric or magnetic fields.

The primary goal of the present work is to describe the motion of a classical electric dipole in the presence of an external magnetic field, perpendicular to the dipole's plane of motion. This system has been approached from various perspective by several authors [4–7]. However, the trajectory of the center of mass and the conditions for the existence of *trapped states* in terms of the constants of motion have not been fully studied. In this article, we describe in detail the solution of the equations of motion in the coordinates of the relative motion and the center of mass, which we derive from the Lagrangian formulation of the problem. The relation between the constants of motion, which permits the existence of trapped states, is established.

As previously discussed, a model of rigid and non-rigid dipoles is considered, constraining the motion of the center of mass to a direction that is perpendicular to the magnetic field [7]. The motion of the relative coordinate into the plane is defined by the direction of the magnetic field and a direction perpendicular to the motion of the center of mass. It is possible to show that for certain values of the characteristic parameters defined in the problem, there is a functional relation between two constants of motion that allows the existence of trapped states [4]; this relation has not yet been analytically established. In other words, an interval of values is found for the constant of motion where solutions are possible and its trend of these solutions is well defined for certain limiting values. These states are called classical bound states embedded in a continuum. The quantum analogue is also discussed [7].

In addition, equations and constants of motion are found for the model of an electric dipole in an external magnetic field, and a preliminary discussion of the existence of trapped states is introduced [4]. In addition, a model of two interacting particles is discussed [6], and special trajectories are found in this model for several initial conditions of the velocity, direction, charges and values of the magnetic field. The distance between particles may vary, but the conditions constrain the motion to a plane perpendicular to the field and to a fixed distance between particles. Furthermore, the classical dynamics of two interacting particles becomes an interesting problem where the challenge is to find solutions that are fully analytical [4–7].

These solutions could significantly impact the future of the applications and construction to molecular motors, as they describe the overall behavior of a dipole from a classical perspective. This paper is organized as follows: In Section 2 we present the theoretical basis of the system, deriving the equations and constants of motions. In Section 3 the solution of the equation of motion is obtained for the center of mass coordinates. In Section 4 we address the conditions that lead to trapped states, as mentioned above. Finally, in Section 5, we offer some concluding remarks.

2. Basic definitions and equations of motion

In the present model [4], we consider two charges in the presence of a uniform magnetic field. The magnetic field is obtained from a vector potential $\vec{A}$, as follows: $\vec{B} = \nabla \times \vec{A}$. We assign to the particle 1(2) the charge $e_1(e_2)$, the position $\vec{r}_1(\vec{r}_2)$, the velocity $\dot{\vec{r}}_1(\dot{\vec{r}}_2)$ and the mass $m_1(m_2)$. The Lagrangian formulation leads to the following expression:

$$L(\vec{r}_1, \vec{r}_2; \dot{\vec{r}}_1, \dot{\vec{r}}_2) = \frac{1}{2} m_1 \dot{\vec{r}}_1^{\,2} + \frac{1}{2} m_2 \dot{\vec{r}}_2^{\,2} - \frac{e_1}{c} \vec{A}(\vec{r}_1) \cdot \dot{\vec{r}}_1 - \frac{e_2}{c} \vec{A}(\vec{r}_2) \cdot \dot{\vec{r}}_2 - \frac{e_1 e_2}{\kappa |\vec{r}_2 - \vec{r}_1|}, \tag{1}$$

where κ is the dielectric constant of the medium in which the motion of charges occurs. We define the vector potential $\vec{A}$ using the symmetric gauge as follows:

$$\vec{A}(\vec{r}_i) = \frac{1}{2} \vec{B} \times \vec{r}_i, \quad \text{for } i = 1, 2, \tag{2}$$