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From superoperator formalism to nonequilibrium Thermo Field Dynamics

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ABSTRACT

Emphasizing that the specification of the representation space or the quasiparticle picture is essential in nonequilibrium quantum field system, we have constructed the unique unperturbed representation of the interaction picture in the superoperator formalism. To achieve it, we put the three basic requirements (the existence of the quasiparticle picture at each instant of time, the macroscopic causality and the relaxation to equilibrium). From the resultant representation follows the formulation of nonequilibrium Thermo Field Dynamics (TFD). The two parameters, the number distribution and excitation energy, characterizing the representation, are to be determined by the renormalization condition. While we point out that the diagonalization condition by Chu and Umezawa is inconsistent with the equilibrium theory, we propose a new renormalization condition as a generalization of the on-shell renormalization on the self-energy which derives the quantum transport equation and determines the renormalized excitation energy.

1. Introduction

A choice of the representation space, or the Fock space, is essential in quantum field theory. Without specifying it, it would be impossible to calculate matrix elements of field operators namely physical quantities. In quantum field theory of vacuum, we can rely on the picture of the stable asymptotic particle on the stable and unique vacuum, and the representation space is the Fock space associated with the asymptotic particles. Because the concept of the asymptotic field is achieved by the

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disappearance of overlap of the wave-packets describing the particles in the infinite past and future, it is clearly invalid in the thermal system. If we insist on the existence of the asymptotic particles in thermal system, the S-matrix is unity (so-called "trivial S-matrix") [1,2]. To this one may add the physical argument that a Hamiltonian for thermal system is not bounded below (see e.g. Eq. (25)) and that a stable particle picture cannot be admitted, which is seen from non-vanishing imaginary part of the self-energy under the thermal situation. Hence every particle becomes thermally unstable and should be called a quasiparticle. What we aim in this paper is to construct the quasiparticle picture explicitly in the unperturbed representation of the interaction picture under thermal situation, which supplants the asymptotic particle picture in the vacuum theory.

There are known two nonequilibrium thermal field theories, i.e., the closed time path (CTP) formalism [3] and Thermo Field Dynamics (TFD) [4]. While the CTP formalism is widely used, we have been employing the TFD formalism because the concept of the representation space is clear even in the nonequilibrium situation. In TFD, which is a real-time canonical formalism of quantum field theory, the thermal degree of freedom is introduced through doubling each degree of freedom, and the mixed state expectation in the density matrix formalism is replaced with an average of a pure state vacuum, called the thermal vacuum. A well-defined quasiparticle picture is constructed on the doubled Fock space, and quasiparticle operators which diagonalize the unperturbed TFD Hamiltonian is defined in a self-consistent manner. A time-dependent number distribution is introduced as an unknown parameter, and the self-consistent renormalization condition [5] derives an equation for it, i.e. the quantum transport equation, which reduces to the well-known quantum Boltzmann equation in the Markovian limit.

So far we have investigated the cold atomic gas systems without and with Bose–Einstein condensate [6–8], and have derived the quantum transport equation in nonequilibrium TFD [9,10]. The appropriate choice of the quasiparticle picture was essential there. As an important result of our previous works, our transport equation in the presence of Bose–Einstein condensate obtains an additional collision term, called the triple production term, which is absent in the other approaches [11–13]. While the triple production term vanishes for the stable condensate, it remains non-vanishing in case of the thermally unstable condensate to prevent the system from equilibrating. Such instability, called the Landau instability and caused by collisions in which the negative-energy quasiparticles participate, has been observed in the cold atomic system [14]. An approach using an inappropriate particle picture, for example the particle picture of original atom with energy spectrum $E(\boldsymbol{x}, \boldsymbol{p}) = p^2/2m + V(\boldsymbol{x})$, takes account of no negative-energy particle and is inadequate to describe the Landau instability. This is an example showing the importance of the choice of the quasiparticle picture.

In this paper we take a position that a representation space corresponding to a particular quasiparticle picture is chosen by taking an unperturbed representation in the interaction picture. This is done in the superoperator formalism [15], following from the density matrix formalism. Then we put the three basic requirements which thermal field theory with the coexistence of microscopic and time-dependent macroscopic quantities is desired to fulfill, and acquire the unique unperturbed representation in which the unperturbed time-dependent density matrix has the geometrical distribution similarly as the equilibrium one. The discussions of this paper is confined to cases of no spontaneous breakdown of symmetry, i.e. without condensates. As will be easily seen, the formulation of nonequilibrium TFD is derived from this representation. It should be emphasized that what we have done in this paper is not a mere re-derivation of nonequilibrium TFD, but we provide a new perspective and understanding of it. First, it is shown that the formulation of nonequilibrium TFD using the thermal vacuum and time-dependent thermal Bogoliubov transformation, which has been assumed without a sound rationale, corresponds to the unique consistent choice of the unperturbed representation. We remark that the Feynman diagram method, a very powerful tool of quantum field theory though it is never guaranteed in thermal field theory for nonequilibrium system, is available owing to it. Second, we point out an undesirable property of the renormalization condition proposed by Chu and Umezawa [5], called the diagonalization condition. Explicitly the diagonalization condition is consistent only in the leading order, since it conflicts with the equilibrium theory in higher orders. In the context of the discussions in the present paper, we find and propose a new self-consistent renormalization condition which overcomes the difficulty. Our renormalization condition reduces to