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Scaling behavior and phase diagram of a $\mathcal{PT}$-symmetric non-Hermitian Bose–Hubbard system

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ABSTRACT

We study scaling behavior and phase diagram of a $\mathcal{PT}$-symmetric non-Hermitian Bose–Hubbard model. In the free interaction case, using both analytical and numerical approaches, the metric operator for many-particle is constructed. The derived properties of the metric operator, similarity matrix and equivalent Hamiltonian reflect the fact that all the matrix elements change dramatically with diverging derivatives near the exceptional point. In the nonzero interaction case, it is found that even small on-site interaction can break the $\mathcal{PT}$ symmetry drastically. It is demonstrated that the scaling law can be established for the exceptional point in both small and large interaction limit. Based on perturbation and numerical methods, we also find that the phase diagram shows rich structure: there exist multiple regions of unbroken $\mathcal{PT}$ symmetry.

1. Introduction

Non-Hermitian Hamiltonian is traditionally used to describe open system phenomenologically. It has profound applications in nuclear physics, quantum transport, quantum chemistry, as well as in quantum optics [1]. Since the discovery of a parity-time ($\mathcal{PT}$) symmetric non-Hermitian Hamiltonian can still have an entirely real spectrum, extensive efforts were paid to the pseudo-Hermitian quantum theory [2–25], which paved the way to our understanding of the connection between non-Hermitian systems and the real physical world. In general, a $\mathcal{PT}$-symmetric non-Hermitian Hamiltonian has unbroken as well as broken $\mathcal{PT}$-symmetric phases, the phase boundary is referred to as the exceptional points (EPs). Studies of the EPs were presented theoretically and experimentally over a decade ago [7–9]. Recently, the experimental realization of $\mathcal{PT}$-symmetric

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systems in optics were suggested through creating a medium with alternating regions of gain and loss [20–22], in which the complex refractive index satisfies the condition $V(x) = V^*(-x)$ and $\mathcal{PT}$ symmetry breaking was observed [23].

One of the characteristic features of the $\mathcal{PT}$-symmetric system is the ubiquitous phase diagram which depicts the symmetry of the eigenfunctions and the reality of the spectrum [11]. The phase separation arises from the fact that although H and the $\mathcal{PT}$ operator commute, the eigenstates of H may or may not be eigenstates of the $\mathcal{PT}$ operator, since the $\mathcal{PT}$ operator is not linear. In the broken $\mathcal{PT}$-symmetric phase the spectrum becomes partially or completely complex, while in the unbroken $\mathcal{PT}$-symmetric phase both H and $\mathcal{PT}$ share the same set of eigenvectors and the spectrum is entirely real. Recently, the phase diagram of a lattice model has been investigated. It is shown that the critical point is sensitive to the distribution of the coupling constant and on-site potential [26–29].

In this paper, we investigate the effect of on-site interaction on the phase boundary of a $\mathcal{PT}$-symmetric Bose–Hubbard system. Our approach is based on our previous work in Ref. [24], where we have systematically investigated an N-site tight-binding chain with a pair of conjugate imaginary potentials $\pm i\gamma$ located at edges. Here we will generalize this description by considering many-particle system and adding the on-site Hubbard interaction U. In the free interaction case, many-particle eigenstates are obtained in aid of the single-particle solutions. We also construct the metric operator to investigate the Hermitian counterpart and observables in the framework of complex quantum mechanics. In nonzero U case, we restrict our attention to the influence of the nonlinear on-site interaction U on the boundary between unbroken and broken $\mathcal{PT}$-symmetric phases. Exact Bethe ansatz solutions and numerical results show that small on-site interaction can reduce the critical point γ_c drastically. Moreover, numerical results show that there exist multiple regions of unbroken $\mathcal{PT}$ symmetry and the number of such regions increases as the system size N increases.

This paper is organized as follows. Section 2 describes the Hamiltonian of a $\mathcal{PT}$-symmetric Bose–Hubbard model. In Section 3, we focus on the interaction-free case. Based on the single-particle solutions, we construct the many-particle eigenstates and metric operator to study the Hermitian counterpart and observables. Section 4 is devoted to the case of nonzero interaction. Based on the approximation solutions, we investigate the critical scaling behavior and the phase diagram. Our findings are briefly summarized and the physical relevance of the model and results are discussed in Section 5.

2. $\mathcal{PT}$-symmetric Bose–Hubbard model

The Bose–Hubbard model gives an approximate description of the physics of interacting bosons on a lattice. Since it embodies essential features of ultracold atoms in optical lattices, the Bose–Hubbard model plays an important role in quantum many-body physics [30,31]. Optical realization of two-site Bose–Hubbard model in coupled cavity arrays and waveguides have been proposed [32–34]. It is worth noting that non-Hermitian Bose–Hubbard dimer has attracted enormous research attention in recent years [35–40]. Theoretical investigations on two site open Bose–Hubbard system was firstly presented in [35]. For a $\mathcal{PT}$-symmetric non-Hermitian Bose–Hubbard dimer, the spectrum and the exceptional points were studied in [36]. After that, dynamics in a leaking double well trap described by non-Hermitian Bose–Hubbard Hamiltonian with additional decay term was investigated under the mean field approximation [37]. Through dynamical study of Bose–Einstein condensed gases, it was shown that imaginary periodic potential may induce perfect quantum coherence between two different condensates [39]. The realization of such open system can be put into practice as a BEC in a double well trap, where the condensate could escape from the traps via tunneling. Most investigations mainly focus on the Bose–Hubbard model with effective decay term in one site. However, it should be noticed that a non-Hermitian Bose–Hubbard dimer with complex coupling terms has also attracted some attention, the decay of quantum states could be controlled by modulating the particle–particle interaction strength and the dissipation in the tunneling process [40]. On the other hand, the Bose–Hubbard model with particle loss was investigated in an alternative way through employing the Lindblad master equation [41], which phenomenologically describes non-unitary evolution of an open system [42]. Recently, $\mathcal{PT}$-symmetric quantum Liouvillean dynamics is also investigated [43]. In