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Landau electron in a rotating environment: A general factorization of time evolution

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ABSTRACT

For the Landau problem with a rotating magnetic field and a confining potential in the (changing) direction of the field, we derive a general factorization of the time evolution operator that includes the adiabatic factorization as a special case. The confining potential is assumed to be of a general form and it can correspond to nonlinear Heisenberg equations of motion. The rotation operator associated with the solid angle Berry phase is used to transform the problem to a rotating reference frame. In the rotating reference frame, we derive a natural factorization of the time evolution operator by recognizing the crucial role played by a gauge transformation. The major complexity of the problem arises from the coupling between motion in the direction of the magnetic field and motion perpendicular to the field. In the factorization, this complexity is consolidated into a single operator which approaches the identity operator when the potential confines the particle sufficiently close to a rotating plane perpendicular to the magnetic field. The structure of this operator is clarified by deriving an expression for its generating Hamiltonian. The adiabatic limit and non-adiabatic effects follow as consequences of the general factorization which are clarified using the magnetic translation concept.

1. Introduction

The study of time-dependent quantum systems has intimate connections with the geometric phase concept [1–3] which has many applications in physics. When the Hamiltonian is time-dependent, the time evolution operator is often nontrivial, i.e., $U(t) \neq \exp(-i\int_0^t H(\tau)d\tau)$, and clarifying its structure is of significance in understanding the dynamics of the system.

One may study $U(t)$ corresponding to a time-dependent Hamiltonian by factorizing it into several operators, each of them is simpler at least in some respects than $U(t)$ itself. A well-known example

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of this is found in the proof of the quantum adiabatic theorem, as presented in standard texts such as Messiah [4]. There the time evolution of a system in a changing environment is constructed as the product of three operators, $U = GDU_\epsilon$, where G is a path dependent geometric operator that brings an initial eigenstate to an instantaneous eigenstate of the Hamiltonian, D is a dynamical operator that only contributes dynamical phase factors to these eigenstates and U_ϵ approaches the identity operator in the adiabatic limit $\epsilon \to 0$. The parameter ϵ determines how fast the Hamiltonian $H[\mathbf{R}(s)] = H[\mathbf{R}(\epsilon t)]$ changes with time t, for a given map from $(s \in)[0, 1]$ to a path in the parameter space M to which $\mathbf{R}$ belongs. For any such fixed map from $[0, 1]$ to M, $1/\epsilon$ provides a time scale which is the total time it takes for $\mathbf{R}(\epsilon t)$ to travel through the given path in M. Another relevant time scale is provided by $\hbar/E_g$, where E_g is the minimum energy gap. The adiabatic limit corresponds to $(\hbar/E_g)/(1/\epsilon) \to 0$, which is made use of in the proof of the adiabatic theorem [4,5]. The path-dependent geometric operator is (assuming the Hamiltonian has non-degenerate eigenstates for all times)

$$G(\mathbf{R}(\epsilon t)) = \sum_m |\psi_m(\mathbf{R}(\epsilon t))\rangle\langle\psi_m(\mathbf{R}(0))|, \tag{1}$$

where $|\psi_m(\mathbf{R}(\epsilon t))\rangle$ is the instantaneous eigenstate of $H[\mathbf{R}(\epsilon t)]$ that satisfies

$$\langle\psi_m(\mathbf{R}(\epsilon t))|\dot{\psi}_m(\mathbf{R}(\epsilon t))\rangle = 0. \tag{2}$$

The expressions for the operators D and U_ϵ also involve the use of eigenstates of the Hamiltonian.

This method of factorizing U using instantaneous eigenstates has certain limitations when dealing with degenerate (including infinitely degenerate) energy eigenstates. For instance, in various Landau systems involving a charged particle in time-dependent electromagnetic fields, the instantaneous energy levels can be highly degenerate. Then there is no known general method for obtaining useful information on G (which contains information on the non-Abelian Berry phase) or U_ϵ using instantaneous eigenstates. However, in a specific problem where the Hamiltonian is given, one may use the algebraic structure of the Hamiltonian without referring to individual eigenstates to directly construct a factorization of U that can then be applied to any representation and the associated eigenstates. From this perspective, there seems to be more problems that can be explored.

This change in perspective also allows us to seek useful factorizations of U not limited by the specific form $U = GDU_\epsilon$, as long as the factorization can help clarify the structure of the total time evolution operator U.

2. The problem and general considerations

The Landau problem is of significance in many areas in physics and its variations (see, for instance, [6–9]) have often been discussed. In this paper our purpose is to study a charged particle in a rotating magnetic field and a confining potential in the direction of the magnetic field. The Hamiltonian is

$$H = \frac{1}{2m}\big(\mathbf{p} - e\mathbf{A}(\mathbf{r}, t)\big)^2 + V(\mathbf{r}\cdot\mathbf{n}(\epsilon t) - L), \tag{3}$$

where

$$\mathbf{A}(\mathbf{r}, t) = \frac{1}{2}B\mathbf{n}(\epsilon t) \times \mathbf{r}. \tag{4}$$

The confining potential $V\big(\mathbf{r}\cdot\mathbf{n}(\epsilon t) - L\big)$ is in the (changing) direction of the magnetic field $B\mathbf{n}(\epsilon t)$ and has equilibrium position at the plane $\mathbf{r}\cdot\mathbf{n}(\epsilon t) - L = 0$ which is perpendicular to $\mathbf{n}(\epsilon t)$. The distance between this plane and the origin of the coordinate system is L. This can be seen as an extension of the usual Landau problem where $\mathbf{n}$ is in a fixed direction.

Here, $V\big(\mathbf{r}\cdot\mathbf{n}(\epsilon t) - L\big)$ is assumed to be of a general form. It can be a harmonic oscillator potential or other types of potentials that in general correspond to nonlinear Heisenberg equations of motion. The complexity of the problem is mainly caused by the coupling between motion in the magnetic field