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Spatial particle correlations in light nuclei. I two-particle systems

P. Mei [1], P. Van Isacker *

GANIL, CEA/DSM–CNRS/IN2P3, BP 55027, F-14076 Caen Cedex 5, France

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ABSTRACT

Expressions for spatial two-particle correlations in an *LS*-coupled basis of the harmonic oscillator are used to display the probability distribution of two identical nucleons as a function of their relative distance and their distance from the center of the nucleus. It is shown that a two-nucleon state in the p shell with total orbital angular momentum $L = 0$ and total spin $S = 0$ contains a di-neutron and a cigar-like component with equal probability. This result can also be proven analytically with the use of angular correlation functions. Scattering of the nucleons from the p shell to other shells leads to the enhancement of the di-neutron configuration. A semi-quantitative application to ^{6}He is presented which shows that the probability of the di-neutron configuration in the ground state is of the order of 60%. The long-term goal of this work is to obtain a geometric insight into the properties of nuclei with several nucleons in a valence shell.

1. Introduction

The radius of nuclei close to the stability line grows on average as the power 1/3 of the mass number *A*. It came therefore as a surprise that for a nucleus as light as ^{11}Li an unusually large value for this quantity was deduced from interaction-cross-section measurements [1]. Subsequently, it was realized that this exceptional spatial extension is due to the weak binding of the two outer neutrons to the ^{9}Li core; the term 'halo' was coined to indicate the phenomenon of one or two particles wandering into a classically forbidden region around a tightly bound core. After the introduction of this concept

* Corresponding author. Tel.: +33 2 31 45 45 65.
E-mail addresses: mei@ganil.fr (P. Mei), isacker@ganil.fr (P. Van Isacker).

1 Tel.: +33 2 31 45 46 04.

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in the study of light nuclear systems in the 1980s, it migrated to atomic and molecular physics, prompting a generic treatment of halo properties in diverse systems [2]. Meanwhile, in nuclear physics, other examples of halo nuclei were found, among which the isotopes ^{6}He and ^{8}He [3], the focus of the first two of the present series of papers.

These experimental studies of neutron-rich light nuclei prompted theoretical work using a variety of approaches, as reviewed by Zhukov et al. [4]. In the case of ^{6}He most of the calculations were of a three-body character (the α particle and two neutrons) and all employed rather sophisticated techniques such as an expansion in hyperspherical harmonics, the coordinate-space Faddeev approach or the two-particle Green's function method. In spite of their complexity these calculations gave rise to a simple geometric picture of ^{6}He, with a ground state divided between a 'di-neutron' and a 'cigar-like' configuration (see Fig. 3 of Ref. [4]).

In this series of papers we explore the geometry of few-particle systems starting from the nuclear shell model formulated in a harmonic-oscillator basis. With applications to the nuclei ^{6}He and ^{8}He in mind, we concentrate in papers I and II on systems with two and four identical particles, respectively. In these first two papers the appropriate background and algorithms are developed, necessary to probe the geometric structure of systems of non-identical particles which will be the subject of the third in the series. Our approach is basic and does not require techniques that go beyond elementary quantum mechanics. It uses in particular the concept of two-particle correlation function which is recalled in Section 2. Within this simple approach it can be shown (Section 3) that the di-neutron and cigar-like configurations mentioned above are an inevitable consequence of the geometry of the p shell, to which the effect of correlations should be added. This property can be derived analytically with use of the *angular* correlation function between the two particles, which will be the more appropriate formulation for the generalization toward more than two particles. A semi-quantitative application of this approach to ^{6}He is considered in Section 4. In Section 5 the conclusions and perspectives of this work are formulated.

2. Two-particle correlation functions

The relevant physical observables that determine the spatial structure of two valence nucleons outside an inert core, such as in ^{6}He, are the relative distance between the two nucleons and the distance between the center of mass of the closed-shell core and that of the two nucleons. These observables can be probed by what will be called two-particle correlation functions. A two-particle correlation function is the expectation value of $\delta(\bar{r} - \bar{r}_{12})$, where $\bar{r}_{12} \equiv \bar{r}_1 - \bar{r}_2$ is the difference between the position vectors of the two particles, $\bar{r}_1$ and $\bar{r}_2$, and $\bar{r}$ is an arbitrary vector. The expectation value of this operator, multiplied with the appropriate volume element, measures the probability of finding the two particles separated by $\bar{r}$. Usually one is only interested in this probability as a function of the distance $r_{12} \equiv |\bar{r}_1 - \bar{r}_2|$ between the two particles which then involves the operator $\delta(r - r_{12})$. Also, it is often useful to probe in addition the distance of the two particles from the center of the nucleus and this can be achieved by calculating the expectation value of $\delta(r - r_{12})\delta(R - R_{12})$, where $R_{12} \equiv |\bar{r}_1 + \bar{r}_2|/2$ is the radial coordinate of the center of mass of the two particles.

These notions can be applied to any quantum-mechanical many-body state by defining the correlation functions

$$\begin{aligned} \mathcal{C}_{\alpha\alpha'}(\bar{r}) &\equiv \left\langle \alpha \left| \sum_{i<j} \delta(\bar{r} - \bar{r}_{ij}) \right| \alpha' \right\rangle, \\ \mathcal{C}_{\alpha\alpha'}(r) &\equiv \left\langle \alpha \left| \sum_{i<j} \delta(r - r_{ij}) \right| \alpha' \right\rangle, \\ \mathcal{C}_{\alpha\alpha'}(r, R) &\equiv \left\langle \alpha \left| \sum_{i<j} \delta(r - r_{ij})\delta(R - R_{ij}) \right| \alpha' \right\rangle, \end{aligned} \tag{1}$$