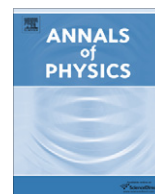




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Three approaches to classical thermal field theory

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ABSTRACT

In this paper we study three different functional approaches to classical thermal field theory, which turn out to be the classical counterparts of three well-known different formulations of quantum thermal field theory: the closed-time path (CTP) formalism, the thermofield dynamics (TFD) and the Matsubara approach.

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1. Introduction

In recent years, there has been a growing interest in the physics of collisions between heavy ions [1,2]. In particular, one of the most interesting topics in the field has become the study of the quark-gluon plasma (QGP), which is a state of baryonic matter characterized by high values of temperature and density and by the fact that quarks are not confined [3]. A better understanding of the physics of this state would be an important step forward in high energy physics and it could have important implications for early universe cosmology as well [4].

The theoretical framework which is required to study the QGP is the finite temperature quantum field theory. In fact, this formalism can take into account the quantum, the relativistic and the statistical features of the system at the same time. There are evidences that in the QGP gluons are characterized by very large occupation numbers. In this state, one would expect the system to be described by a classical field theory [5]. In fact, it is known that QED is well-approximated by the classical Maxwell theory when occupation numbers of photons become large. Analogously, when occupation numbers of gluons are large, it should be possible to approximate QCD with a classical gauge field theory [6].

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These considerations have led various authors to study classical field theories at high temperatures and, to our knowledge, three main approaches have been pursued in the literature so far. The first approach is due to Aarts and collaborators [7–11], who computed classical thermal correlation functions $\langle \phi(x) \cdots \phi(x') \rangle_\beta$ by solving the Hamiltonian equations of motion and averaging the solutions over a canonical ensemble of initial conditions. An alternative approach has been proposed by Cooper et al. [12] and it is based on the operatorial formalism developed by Martin et al. [13] in the 1970s and known as MSR formalism. Finally, Jeon [14] has recently studied a classical scalar field theory at finite temperature by resorting to the path integral approach to classical mechanics proposed in Ref. [15] at the end of the 80s.

Despite the formal differences, there is a common feature shared by the three classical approaches mentioned above: they are all related to the quantum closed-time path (CTP) formalism [16,17] in the high temperature limit. However, quantum field theory at finite temperature admits several other equivalent formulations, the most popular ones being the Matsubara formalism [18] and the thermofield dynamics (TFD) [19]. The goal of this paper is to study the “classical analogs” of the TFD and the Matsubara formalism as well. To this end, we will present a general framework for classical thermal field theory based on the operatorial approach to classical mechanics proposed in the 30’s by Koopman and von Neumann (KvN) [20–22] and on the associated classical path-integral (CPI) formulation [15]. We believe that these tools (KvN and CPI) are the most suitable ones for studying the interplay between classical and quantum mechanics [23] either with or without temperature. Within this theoretical framework, we will show how the *classical* finite temperature field theory can be formulated in three different ways, which are in a certain sense the “classical counterparts” of the three *quantum* formalisms mentioned above: the CTP, the TFD and the Matsubara one. All these formalisms will be illustrated by considering the simple example of a scalar field with quartic self-interaction.

The paper is organized as follows: after reviewing briefly the various equivalent approaches to quantum thermal field theory (Section 2) and the KvN and CPI formulations of classical mechanics (Section 3), in Section 4 we will present the approach to classical thermal field theory developed in [14]. From a physical point of view, this is the most intuitive approach to classical thermal field theory, as it is based on the same principles used by Aarts and Smit in [7,8], i.e. on solving the equations of motion and averaging over the initial conditions using the canonical distribution. As we already mentioned, this approach turns out to be related to the CTP approach to quantum thermal field theory in the high temperature limit.

Then, in the second part of the paper we shall develop two new approaches to *classical* thermal field theory which display both a remarkable formal analogy and a quantitative agreement with their *quantum* counterparts at high temperatures, i.e. the TFD approach and the Matsubara formalism. In particular, in Section 5 we will study the TFD approach to *classical* thermal field theory, which is based on the idea of implementing thermal averages as expectation values on a particular state $|\psi_\beta\rangle$ in the *classical* Hilbert space of Ref. [24]. In Section 6 we will study the classical counterpart of the quantum Matsubara formalism. Both quantum and classical Matsubara approaches allow us to compute only static properties of system, since the time variable is formally restricted to the purely imaginary axis. However, while in the quantum case time is an element of a complex plane, in the classical case time is an element of a complex *superspace* [25]. As we will see, the appearance of superspace is just a natural consequence of the fact that the path integral formulation of classical mechanics features an universal $N = 2$ supersymmetry and, as such, admits a very compact representation in terms of some suitably defined superfields [23]. Finally, some further background material as well as some technical details were included in two appendices.

2. Quantum thermal field theory

In order to make the paper as self-contained as possible, in this section we will review some aspects of quantum field theory at finite temperature. For a more exhaustive treatment we refer the reader to the review articles [26,27] or the textbooks [28,29]. The main goal of this section is to show how perturbative calculations can be implemented in several different ways, and to derive the

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