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Unifying treatment of nonequilibrium and unstable dynamics of cold bosonic atom system with time-dependent order parameter in Thermo Field Dynamics

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ABSTRACT

The coupled equations which describe the temporal evolution of the Bose–Einstein condensed system are derived in the framework of nonequilibrium Thermo Field Dynamics. The key element is that they are not the naive assemblages of assumed equations, but are the self-consistent ones derived by appropriate renormalization conditions. While the order parameter is time-dependent, an explicit quasiparticle picture is constructed by a time-dependent expansion. Our formulation is valid even for the system with a unstable condensate, and describes the condensate decay caused by the Landau instability as well as by the dynamical one.

1. Introduction

The systems of trapped cold atoms are ideal for studying the foundations of quantum many-body theories such as quantum field theory and thermal field theory. They are dilute and weak-interacting, so theoretical calculations can be compared with experimental results directly. Since the realization of Bose–Einstein condensates [1–3], many intriguing phenomena have been observed with good accuracy, and offer opportunities to test many aspects of quantum many-body theories in both equilibrium and nonequilibrium. Among them, the unstable phenomena of the condensate attract our attention, because firstly to formulate unstable quantum many-body systems is still an open problem and secondly nonequilibrium processes accompany the instability in thermal situation.

Theoretically, the instability of the condensate is characterized by the eigenvalue of the Bogoliubov–de Gennes (BdG) equations [4–6], which follow from linearization of the time-dependent

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Gross–Pitaevskii (TDGP) equation [7]. Since the BdG equations are generally eigenvalue equations for non-Hermitian matrices, their eigenvalues can be complex. The emergence of complex eigenvalues is interpreted as the sign of the dynamical instability. This instability is associated with the decay of the initial configuration of the condensate and can occur even at zero temperature. On the other hand, if the negative eigenvalues for a positive-norm mode are present, the system shows other instability, called the Landau instability. This instability, in which the thermal cloud plays an essential role to drive the condensate toward a lower energy state, is suppressed at very low temperature.

The observations of both the Landau and dynamical instabilities are reported in several systems, especially in the system where the condensate flows in an optical lattice [8,9], and they are in good agreement with the analyses of the BdG equations [10,11].

Although the TDGP equation outlines the experiments corresponding to the dynamical instability at very low temperature [12,13], e.g., the multiply-quantized vortex splitting [14], a detailed description of the unstable dynamics in thermal situations is not trivial. That is because there is no more quasi-stable state, and so a fully nonequilibrium theory is required.

There are two known nonequilibrium thermal field theories, i.e., the closed time path (CTP) formalism [15] and Thermo Field Dynamics (TFD) [16]. TFD is a real-time canonical formalism of quantum field theory in which thermal fluctuation is introduced through doubling the degrees of freedom, and the mixed state expectation in the density matrix formalism is replaced by an average of a pure state vacuum, called the thermal vacuum. In the canonical formalism including TFD, the representation space (Fock space) to represent the field operators is constructed explicitly in the interaction picture, and its choice corresponds to that of the quasiparticle picture. We note that the choice of the representation space is critical in case of the spontaneous breakdown of symmetry such as the Bose–Einstein condensation. The quasiparticle picture in a nonequilibrium process with a time-dependent Bose–Einstein condensate should change in time, as will be studied in this paper. While the quasiparticle picture is specified only at an initial time and its temporal change is implicit in CTP, TFD is designed to follow its temporal change. We consider that time-dependence of the quasiparticle picture is a very important element in the nonequilibrium process, and that TFD is suited for the purpose of describing it, better than CTP.

In our previous paper [17], we have investigated the cold atom system with a time-independent configuration of the condensate in TFD, and derived the quantum transport equation which describes the temporal evolution of the quasiparticle number distribution. It was essential to construct an explicit quasiparticle picture there. In contrast to the previous investigations [18–22] which are based on a phase-space distribution function, our transport equation contains an additional collision term which is traced back to our choice of an appropriate quasiparticle picture. The additional collision term, which we call the triple production term, vanishes in the equilibrium limit if there is no Landau instability, but remains non-vanishing to prevent the system from equilibrating if there is Landau instability. Thus our transport equation with the triple production term and the other ones without it predict definitely different behaviors of the unstable system.

In this paper, we extend the study of a stationary condensate [17] to a case of time-dependent condensate. As was mentioned above, this extension involves temporal change of the quasiparticle picture and is not simple but drastic. We derive the coupled equations which describe the nonequilibrium dynamics of the cold atom system with a time-dependent order parameter. They are the TDGP equation, the time-dependent Bogoliubov–de Gennes (TDBdG) equations, and the quantum transport equation. Our scenario is as follows: while the order parameter is time-dependent, the field operators are expanded in the time-dependent complete set of the solutions of the TDBdG equations [23], which characterize the excitation modes. We prepare a stable thermal vacuum and introduce a time-dependent thermal Bogoliubov transformation. The quantum correction to the TDGP equation is determined self-consistently and simultaneously as the quantum transport equation by some renormalization conditions [24]. Solving the coupled equations numerically, we illustrate the dynamics of the condensate decays with either the Landau instability or the dynamical one and discriminate the two instabilities.

This paper is organized as follows. We consider the cold bosonic atom system with a time-dependent order parameter at zero temperature in Section 2. We show that it is crucial to expand the field operator by the solutions of TDBdG equations to maintain the time-dependent quasiparticle picture. In