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Collective modes in asymmetric ultracold Fermi systems

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ABSTRACT

We derive the long wavelength effective action for the collective modes in systems of fermions interacting via a short-range *s*-wave attraction, featuring unequal chemical potentials for the two fermionic species (asymmetric systems). As a consequence of the attractive interaction, fermions form a condensate that spontaneously breaks the $U(1)$ symmetry associated with total number conservation. Therefore at sufficiently small temperatures and asymmetries, the system is a superfluid. We reproduce previous results for the stability conditions of the system as a function of the four-fermion coupling and asymmetry. We obtain these results analyzing the coefficients of the low energy effective Lagrangian of the modes describing fluctuations in the magnitude (Higgs mode) and in the phase (Nambu–Goldstone, or Anderson–Bogoliubov, mode) of the difermion condensate. We find that for certain values of parameters, the mass of the Higgs mode decreases with increasing mismatch between the chemical potentials of the two populations, if we keep the scattering length and the gap parameter constant. Furthermore, we find that the energy cost for creating a position dependent fluctuation of the condensate is constant in the gapped region and increases in the gapless region. These two features may lead to experimentally detectable effects. As an example, we argue that if the superfluid is put in rotation, the square of the radius of the outer core of a vortex should sharply increase on increasing the asymmetry, when we pass through the relevant region in the gapless superfluid phase. Finally, by gauging the global $U(1)$ symmetry, we relate the coefficients of the effective Lagrangian of the Nambu–Goldstone mode with the screening masses of the gauge field.

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1. Introduction

Experiments with trapped cold atomic gases have driven a renewed interest in fermionic pairing [1,2]. In particular, much effort has been devoted to understanding the superfluid phases of imbalanced fermionic gases, featuring unequal number of particles of the distinct fermionic species that pair [3–24].

The system consists of fermions of two different species, ψ_1 and ψ_2, which correspond to two hyperfine states of a fermionic atom like ^{6}Li. These fermions have opposite spin and the interaction between them can be tuned by employing a Feshbach resonance [25]. The strength of the interaction is given in terms of the *s*-wave scattering length between the two species.

For zero imbalance, the system properties are qualitatively well understood using mean field theory [26]. In weak coupling the system lives in a weakly coupled BCS state and crosses over to a strongly coupled BEC state through the resonance region. While the extreme BCS and BEC regimes are also in good quantitative control in mean field theory, close to resonance (the unitarity region) a quantitative understanding of the phases comes mainly from Monte-Carlo calculations [11]. (For other approaches see [27–29].) This is because close to resonance the scattering length is much larger than the inter-particle distance and there is no small parameter in the Lagrangian to expand in. Therefore fluctuations may change the mean field results substantially.

In standard BCS superfluids the chemical potentials of the two fermionic species are equal. An imbalance in the number of ψ_1 and ψ_2 is implemented by taking the chemical potentials for the two species, μ_1 and μ_2 respectively, to be different. (We will name our species in a way that $\mu_1 \geq \mu_2$.) If the chemical potential difference, $2\delta\mu = \mu_1 - \mu_2$ is much smaller than the magnitude of the gap parameter $|\Delta|$, the splitting cannot disrupt BCS superfluidity because the superfluid state with equal number densities is energetically favored in comparison with a normal state with a fermionic imbalance. On the other hand, as pointed out in [3], in the weak coupling regime, BCS superfluidity cannot persist for large values of $\delta\mu$. Indeed, there exists an upper limit for $\delta\mu$ (the so-called Chandrasekhar–Clogston limit), beyond which the homogeneous superfluid state is no longer energetically favored over the normal phase.

For imbalanced systems, a qualitatively complete picture of the phase diagram has not been established yet. Proposed possibilities are phase-separation [7], breached pair superfluidity [4,8–10], deformed Fermi sea pairing [6] and non-homogeneous or LOFF pairing [5]. (See [30,31] for reviews.)

The phase diagram of the system at $T = 0$ as a function of the scattering length and the chemical potential difference has been explored in the mean field approximation in [13,16,23,32]. The authors find that on the BCS side of the resonance there are no stable homogeneous superfluid phases that have gapless Fermi surfaces. On the BEC side of the resonance, there are stable gapless superfluid phases, which can exhibit a net polarization. At resonance, mean field theory suggests a first order phase transition from the superfluid to the normal phase as $\delta\mu$ is increased, without any intervening gapless superfluid phase. Consequences of the phase diagram for experiments with trapped atoms were explored in [16,18]. At resonance if we fill different numbers of ψ_1 and ψ_2 in the harmonic trap, because the gapped phase cannot feature a net polarization, the system phase separates with an unpolarized superfluid in the central region of the trap and a polarized normal fluid at the exterior.

For non-zero imbalance close to the resonance, fluctuations may change the mean field results qualitatively. This has to be contrasted with the zero imbalance case, where fluctuations lead only to a quantitative change of the mean field results. Indeed for non-zero imbalance many features of the phase diagram are not caught by the mean field approximation. The authors of [15] go beyond mean field theory by using results from Monte-Carlo simulations [24] and propose a phase diagram which features a splitting point near resonance at non-zero $\delta\mu$, where the homogeneous superfluid, a LOFF-like inhomogeneous phase, and the gapless superfluid phase coexist. They also find stable gapless fermionic modes with one and two Fermi surfaces, on the BCS side of the resonance. A detailed treatment of fluctuations around the resonance using an expansion in $\epsilon = D - 4$ space dimensions at $T = 0$ [29,33] supports this picture. A different approach consists in generalizing the Fermi gas to a model with *2N* hyperfine states, performing a systematic $1/N$ loop expansion around the BEC–BCS solution [27,34]. The phase diagram at unitarity has also been explored using a Superfluid Local Density Approximation (SLDA) [35,36]. With this method one finds that on increasing $\delta\mu$ from zero at unitarity, there is an intervening window of values for which the LOFF phase is favored over the homogeneous superfluid and the normal phases.

In this paper, we study small fluctuations about the mean field value of the gap parameter for a system with mismatched Fermi surfaces. We consider fluctuations of Δ both in its phase and in its magnitude. Both