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2PI functional techniques for gauge theories: QED

Urko Reinosa [a,*,1], Julien Serreau [b,2]

[a] Centre de Physique Théorique École Polytechnique, CNRS, 91128 Palaiseau, France
[b] Astro-Particule et Cosmologie, Université Paris 7 – Denis Diderot, 10, rue A. Domon et L. Duquet, 75205 Paris Cedex 13, France

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We discuss the formulation of the prototype gauge field theory, QED, in the context of two-particle-irreducible (2PI) functional techniques with particular emphasis on the issues of renormalization and gauge symmetry. We show how to renormalize all *n*-point vertex functions of the (gauge-fixed) theory at any approximation order in the 2PI loop-expansion by properly adjusting a finite set of local counterterms consistent with the underlying gauge symmetry. The paper is divided in three parts: a self-contained presentation of the main results and their possible implementation for practical applications; a detailed analysis of ultraviolet divergences and their removal; a number of appendices collecting technical details.

1. Introduction

Two-particle-irreducible (2PI) functional techniques [1,2] provide a powerful tool to generate systematic partial resummations of perturbative series in (quantum) field theory, of interest in numerous physical situations where the perturbative expansion breaks down. Topical examples include thermodynamic [3–6] and transport [7] properties of bosonic fields at high temperatures or at a second order phase transition [8,9], or genuine nonequilibrium situations [10–12], with physical applications ranging from early-universe cosmology [13] to high-energy heavy-ion collisions experiments [14] and condensed matter physics [15]. The application of such techniques to gauge theories is a nontrivial issue which requires one to understand the interplay between renormalization and symmetry. Important progress has been made recently concerning the issue of symmetries in the 2PI formalism, in

* Corresponding author.
E-mail addresses: urko.reinosa@cpht.polytechnique.fr (U. Reinosa), julien.serreau@apc.univ-paris7.fr (J. Serreau).
[1] CPHT is unité mixte de recherche UMR7644 (CNRS, École Polytechnique).
[2] APC is unité mixte de recherche UMR7164 (CNRS, Université Paris 7, CEA, Observatoire de Paris).

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particular, in the context of abelian gauge theories [16]. Moreover, the basic ideas concerning the renormalization of 2PI QED have been put forward in Ref. [17]. We present here a complete renormalization theory of QED in covariant linear gauges in the 2PI framework.

It is known that linearly realized global symmetries of a given theory are inherited by the corresponding 2PI-resummed effective action [12,18]. This has recently been shown to be true also for abelian gauge theories in linear gauges [16]. In particular, it has been shown how to construct 2PI approximation schemes which systematically respect the corresponding Ward-Takahashi identities. Another important issue regarding gauge symmetry concerns the gauge-fixing independence of physical observables [19,20]. The point is that, even though the latter, as defined in the exact theory, are gauge-fixing independent, the corresponding expressions obtained from a given 2PI approximation, contain residual gauge-fixing dependences due to the resummation of infinite subclasses of perturbative contributions inherent to 2PI techniques. It is to be emphasized that the fact that vertex functions obtained from a given 2PI approximation exactly satisfy the correct symmetry identities does not guarantee that the corresponding physical observables are gauge-invariant, i.e. gauge-fixing independent.

For systematic approximation schemes, such as e.g. the 2PI loop-expansion, it can be shown that these gauge dependent contributions are parametrically suppressed in powers of the expansion parameter [19] (see also [4]). The observed good convergence properties of 2PI approximations schemes [5,6] therefore suggests that they should be under control, at least in weakly coupled situations, provided that renormalization is well understood. This has recently been tested for QED in Ref. [21], where the thermodynamic pressure has been computed at finite temperature from the two-loop approximation of the 2PI effective action in covariant gauge. The results indicate that gauge-fixing parameter dependences remain under control in a wide range of couplings and are comparable with the renormalization scale dependences, other sources of uncertainty inherent to such calculations. Moreover, the Landau gauge has been identified as a gauge minimizing both gauge-fixing parameter and renormalization scheme dependences.

The second aspect mentioned above, renormalizability, is deeply related to that of symmetries. Renormalization in the 2PI framework has been understood only in recent years and has been extensively developed for theories with scalar [22,18,5,23,24,9], but also fermionic [25] and, recently, gauge [17,21] degrees of freedom. Most studies have been concerned with the issue of renormalizing the basic equation of the 2PI formalism, that is the self-consistent equation for the two-point function. In Ref. [24] a complete description of all n-point vertex functions in the 2PI framework has been put forward for scalar field theories. The present paper develops the renormalization theory of 2PI QED in covariant linear gauge, extending our previous work [17] for the renormalization of the photon and fermion two-point functions. In particular, we show that all proper vertex functions obtained from the 2PI loop-expansion of QED can be made finite at any approximation order by adjusting a finite set of local counterterms consistent with the underlying gauge symmetry.

In Section 2 we review the 2PI formulation of QED. In particular, we consider the field expansion of the so-called 2PI-resummed effective action, which defines the (2PI-resummed) proper vertex functions of the theory. These can be expressed in terms of a second class of vertex functions which we call 2PI vertices and which require independent renormalization. Section 3 introduces the counterterms needed to renormalize both 2PI and 2PI-resummed vertex functions in a way that preserves the symmetries of the bare theory. The presence of two different classes of vertices and the fact that gauge symmetry constraints them differently [16] call for introducing additional counterterms as compared to the standard (i.e. perturbative) renormalization theory. Still, the number of independent renormalization conditions which define the theory is the same as usual, as we discuss in Section 3.5. Indeed, the extra-counterterms are fixed by imposing consistency conditions between 2PI and 2PI-resummed vertex functions at the renormalization point. This is a standard feature of 2PI renormalization theory [24]. Another important issue, which we discuss in Sections 3.4 and 3.5, is to make sure that renormalization and consistency conditions respect the underlying gauge symmetry. In Section 3.6, we provide a ready-to-use renormalization procedure which allows one to fix all the counterterms in a way that preserves the corresponding 2PI Ward-Takahashi identities. Readers interested in practical applications will find everything they need in those sections.