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$\mathcal{N} = 1$ conformal superspace in four dimensions

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ABSTRACT

We construct in detail an $\mathcal{N} = 1,\ D = 4$ superspace with the superconformal algebra as the structure group and discuss its relation to prior component approaches and the existing Poincaré superspaces.

1. Introduction

The use of conformal techniques to address supergravity has a long history. Not all that long after Wess and Zumino discovered the superspace formulation of supergravity [1], Kaku, Townsend, and van Nieuwenhuizen, along with Ferrara and Grisaru, worked out the conformal structure of component supergravity and demonstrated that Poincaré supergravity was a gauge-fixed version of conformal supergravity [2]. Howe first proposed superspace formulations of four-dimensional $\mathcal{N} \leqslant 4$ conformal supergravities by explicitly gauging $SL(2, \mathcal{C}) \times U(\mathcal{N})$ [3]. Work continued on conformal supergravity over the next few years (an excellent review [4] on the topic was written by Fradkin and Tseytlin) eventually culminating in the work of Kugo and Uehara, who not only popularized the conformal compensator approach to supergravity and matter systems [5] but also made a comprehensive analysis of the component transformation rules and spinorial derivative structure of $\mathcal{N} = 1$ conformal supergravity [6].

In large part, the results presented here are a superspace response to this last work. Here we will take a complementary approach, treating superspace as an honest supermanifold with a conformal structure. Unlike Howe, we will seek to gauge the *entire* superconformal algebra. Prior experience with superspace hints that this approach would be a foolish one – that the constraints required with a

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larger structure group would be more numerous and their evaluation more cumbersome. What we find is the opposite: the covariant derivatives of conformal supergravity have a Yang–Mills structure, with the algebra

$$\begin{aligned}
&\{\nabla_\alpha, \nabla_\beta\} = 0, \quad \left\{\nabla_{\dot\alpha}, \nabla_{\dot\beta}\right\} = 0 \\
&\{\nabla_\alpha, \nabla_{\dot\alpha}\} = -2i\nabla_{\alpha\dot\alpha} \\
&\{\nabla_\beta, \nabla_{\alpha\dot\alpha}\} = -2i\epsilon_{\beta\alpha}\mathcal{W}_{\dot\alpha}, \quad \left\{\nabla_{\dot\beta}, \nabla_{\alpha\dot\alpha}\right\} = -2i\epsilon_{\dot\beta\dot\alpha}\mathcal{W}_\alpha
\end{aligned}$$

where $\mathcal{W}_\alpha$ are the "gaugino superfields" for the superconformal group. The constraints of conformal superspace involve setting most of the $\mathcal{W}_\alpha$ to zero, and the evaluation of these constraints is no more difficult than in a conventional Yang–Mills theory, leading the non-vanishing $\mathcal{W}_\alpha$ to be expressed in terms of the single superfield $W_{\alpha\beta\gamma}$. When the theory is "degauged" to a $U(1)$ Poincaré supergravity, the extra gauge superfields can be reinterpreted as the familiar superfields R, G_c, and X_α. This is the main result of this work.

It is well known that the various equivalent formalisms of superspace supergravity – the minimal Poincaré [7], the minimal Kähler [8], and even the new minimal Poincaré [9] – are all derivable from a conformal superspace under different gauge-fixing constraints. We review one way of seeing how this occurs in our approach.

This paper is divided into two sections. In the first, we discuss conformal representations of superfields on superspace and construct the constraints necessary for the existence of such a space. We also give the explicit form of all the curvatures from solving the Bianchi identities. In the second, we demonstrate how the auxiliary structure of $U(1)$ superspace is identical to a certain gauge-fixed version of conformal superspace. In addition, we explicitly construct the superspace of minimal supergravity, Kähler supergravity, and new minimal supergravity. Included in the appendix is an elementary review of the structure of global and local spacetime symmetry groups as well as the structure of actions over both the full manifold and submanifolds of such theories.

Throughout this paper we use the superspace notations and conventions of Binetruy et al. [8] (which are a slight modification of those of Wess and Bagger [10]) – with our own slight modification: we choose the superspace $U(1)$ connection to be Hermitian. That is, our connection A_M here is equivalent to $-iA_M$ of [8]; similarly, our corresponding generator A is equivalent to their iA. (The unfortunate coincidence of the generator and connection names will, we hope, not overly confuse the reader.)

Although the theory discussed here ought to be properly denoted "superconformal superspace," this is an awkward term that we would like to avoid. Instead we use "conformal" when the subject is superspace. (Similarly, supertranslations on superspace are simply called translations.) When the component theory is under consideration, we restore the "super."

2. Conformal superspace

In the ensuing section, we describe the gauge structure, geometric constraints, and curvatures of conformal superspace, which is defined as a normal $\mathcal{N} = 1$ superspace with the structure group of the superconformal algebra. We discuss representations of that algebra, invariant actions and chiral submanifold actions. As usual, constraints must be imposed to eliminate unwanted fields; we will discuss their construction and solution. But the first place to start is at the component level, where conformal supergravity is well known and its properties well-established.

Some use will be made throughout of results presented in the appendix. Specific references will be made when especially relevant.

2.1. Conformal supergravity at the component level

Conformal supergravity at the component level begins with the extension of the Poincaré to the super-Poincaré algebra by the addition of fermionic internal symmetries Q_α. These anticommute to give spacetime translations: