Annals of Physics 325 (2010) 579–595

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Casimir stress in an inhomogeneous medium ☆

T.G. Philbin [a,*], C. Xiong [b], U. Leonhardt [a]

[a] School of Physics and Astronomy, University of St. Andrews, North Haugh, St. Andrews, Fife KY16 9SS, Scotland, United Kingdom
[b] School of Computer Science, University of St. Andrews, North Haugh, St. Andrews, Fife KY16 9SX, Scotland, United Kingdom

A R T I C L E I N F O

Article history:
Received 16 September 2009
Accepted 21 November 2009
Available online 27 November 2009

Keywords:
Casimir effect
Quantum vacuum
Lifshitz theory
Inhomogeneous media

A B S T R A C T

The Casimir effect in an inhomogeneous dielectric is investigated using Lifshitz's theory of electromagnetic vacuum energy. A permittivity function that depends continuously on one Cartesian coordinate is chosen, bounded on each side by homogeneous dielectrics. The result for the Casimir stress is infinite everywhere inside the inhomogeneous region, a divergence that does not occur for piece-wise homogeneous dielectrics with planar boundaries. A Casimir force per unit volume can be extracted from the infinite stress but it diverges on the boundaries between the inhomogeneous medium and the homogeneous dielectrics. An alternative regularization of the vacuum stress is considered that removes the contribution of the inhomogeneity over small distances, where macroscopic electromagnetism is invalid. The alternative regularization yields a finite Casimir stress inside the inhomogeneous region, but the stress and force per unit volume diverge on the boundaries with the homogeneous dielectrics. The case of inhomogeneous dielectrics with planar boundaries thus falls outside the current understanding of the Casimir effect.

☆ The problem of the Casimir stress in an inhomogeneous medium was suggested by UL. UL and TP developed the general expression for the Green tensor in 1D-inhomogeneous dielectrics. TP and CX investigated simple models and the specific case solved in the paper was chosen by TP. TP and CX independently computed the Casimir stress, using different software packages. The derivation of the analytic solutions for the Green functions was prepared by TP based on the computer solutions by CX and TP. CX found the intrinsic constant problem in the Lifshitz regularization. The new regularization method was developed by UL. The results for the Casimir force per unit area using the conventional regularization were obtained by TP. The figures and plots in the paper were prepared by TP, who wrote the text, with an addition by UL.

* Corresponding author.
E-mail address: tgp3@st-andrews.ac.uk (T.G. Philbin).

0003-4916/$ - see front matter
doi:10.1016/j.aop.2009.11.006

1. Introduction

Quantization of the electromagnetic field leads to a vacuum, zero-point energy-momentum that diverges [1]. As this infinite vacuum energy clearly does not exist it was initially assumed that the infinity should be zero, until Casimir showed [2] that a finite vacuum energy can sometimes be extracted, leading to the prediction of measurable forces.

In recent years there has been considerable progress in the experimental demonstration of Casimir forces between objects [3], and a growing appreciation of the problems and opportunities these forces entail for micro- and nano-engineering [4,5]. On the theory side, it is understood how to calculate the Casimir forces between an arbitrary number of separated bodies, the most versatile approach being that of Lifshitz [6–8] wherein the electromagnetic stress tensor and energy density are obtained from the Green tensor for the electric field (or vector potential). Although the calculations for any arrangement of objects beyond the classic case of two parallel half-spaces [2,6] are extraordinarily cumbersome, the problem can be solved purely numerically using standard codes for the Green tensor [9–11].

Yet the theoretical understanding of the Casimir effect remains incomplete in some fundamental respects. Most importantly, the Casimir self-energy of an object is a quantity whose exact meaning is still debated and for which the standard approaches like Lifshitz theory give a diverging answer in general [3]. The subject of Casimir self-energy began in 1968 with Boyer's spectacular conclusion that the self-energy of an infinitesimally thin, perfectly conducting spherical shell is positive, giving a repulsive (i.e. outwardly directed) Casimir force on the shell [12]. An unsettling feature of the spherical shell is the occurrence of an additional divergence which is not present in the case of perfectly conducting parallel plates [2] and which must be regularized to obtain a finite answer. It might be hoped that the extra divergence is due to the idealized properties of vanishing thickness and perfect conduction of the shell, but the vanishing thickness actually gives rise to a cancellation of some diverging quantities, and the apparently reasonable case of spherically symmetric materials with general electric permittivity and magnetic permeability is even more ill-behaved. Even with allowance for (temporal) dispersion, the additional divergences remain and there is no agreement on their precise significance, or how to remove them [13–21,3]. Hence the striking fact that the seemingly innocuous issue of the Casimir self-energy of a dielectric ball is still not understood [15,3]. Similar problems occur for cylindrically symmetric media, the other case where Casimir self-forces have been investigated in some detail. (See [3] for a guide to the Casimir literature on spherical and cylindrical geometries.) This lack of understanding of the self-force for these geometries makes any attempt to compute the Casimir forces on concentric spherical and cylindrical bodies problematic.

The work described above has all been for piece-wise homogeneous media, with the non-zero Casimir forces arising from the boundaries between the homogeneous components. Consideration of the electrostrictive contribution to the Casimir effect [22,23] introduces some inhomogeneity in the material since it gives rise to an extra surface force that influences the interior of the body. Divergences in the ground-state energy of a smooth dielectric were investigated using quantum field theory in [24]. But despite the fact that the general theory of Lifshitz and his co-workers [6–8] is formulated for inhomogeneous, dispersive dielectrics, to our knowledge there is no example in the literature where the theory is used to calculate the Casimir effect for an inhomogeneous dielectric.[1] In this paper we provide such an example by using Lifshitz theory to calculate the vacuum electromagnetic stress tensor for a simple model of an inhomogeneous dielectric. The stress tensor specifies the local Casimir self-force in the medium arising from its inhomogeneity. We choose the simplest example where the inhomogeneity is in one spatial dimension only, the medium being homogeneous in planes orthogonal to this direction. Boundaries are imposed on both sides of the inhomogeneous medium, beyond which the material is homogeneous. As the boundaries are planar, we expected to avoid the problems with divergences described above, which appear to be associated with curved boundaries and a high degree of symmetry. Our results did not meet these expectations. Specifically, we find that the regularization prescription for inhomogeneous dielectrics advocated in standard Lifshitz theory [7,8] yields an infinite Casimir stress

[1] A previous study [25] considered layers of homogeneous slabs as an approximation to an inhomogeneous dielectric and computed the Casimir energy arising from the TM evanescent modes of the system.