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Decoupling structure of the principal sigma model–Maxwell interactions

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ABSTRACT

The principal sigma model and Abelian gauge fields coupling is studied. By expressing the first-order formulation of the gauge field equations an implicit on-shell scalar–gauge field decoupling structure is revealed. It is also shown that due to this decoupling structure the scalars of the theory belong to the pure sigma model and the gauge fields sector consists of a number of coupled Maxwell theories with currents partially induced by the scalars.

1. Introduction

The scalar sectors of the supersymmetric field theories especially the supergravity theories which are the theories that govern the massless sector low energy background coupling of the relative superstring theories [1] can be formulated as principal sigma models or non-linear sigma models whose target spaces are group manifolds or coset spaces. In particular a great majority of the supergravity scalar sectors are constructed as symmetric space sigma models [2–8]. When the Abelian (Maxwell) vector multiplets are coupled to the graviton multiplets in these theories the scalar sector which has the non-linear sigma model interaction with in itself is coupled to the Abelian gauge fields through a kinetic term in the Lagrangian [9–12].

In this work, we will focus on the gauge field equations of the principal sigma model and the Abelian gauge field couplings mentioned above. Bearing in mind that the non-linear sigma model can be obtained from the principal sigma model by imposing extra restrictions for the sake of generality we will consider the generic form of the principal sigma model as the non-linear interaction of the scalars. We will simply show that the gauge field equations can be locally integrated so that they can be expressed as first-order equations containing arbitrary locally exact differential forms. The first-order form of the gauge field equations will be used to show that there exists a one-sided decoupling between the scalars and the gauge fields in a sense that the scalar field equations do

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not contain the gauge fields in them whereas the scalars enter as sources in the gauge field equations. Thus the scalar solution space of the coupled theory coincides with the general solution space of the pure sigma model. Furthermore we will also discuss that this hidden on-shell scalar-matter decoupling results in a number of coupled Maxwell theories with sources whose currents contain the general solutions of the principal sigma model which is completely decoupled from the Maxwell sector. Therefore we will show that when the general solutions of the pure principal sigma model are obtained and when one fixes the sector of the solution space of the coupled theory by fixing the field dependence or the independence of the locally exact differential forms appearing in the first-order gauge field equations one may determine the currents of the coupled Maxwell fields. In this respect one may solve the gauge fields from the Maxwell sector field equations. Consequently the solution space of the scalar-matter coupling can be entirely generated by the general solutions of the pure principal sigma model and the arbitrary choice of the locally exact differential forms. This fact is a consequence of the solution methodology which is based on the implicit on-shell decoupling between the matter fields and the scalar sector which provides current sources to the former.

2. Hidden decoupling structure of the gauge fields and their sources

In a D-dimensional spacetime M the Lagrangian which gives the inhomogeneous Maxwell equations can be given as

$$\mathcal{L} = -\frac{1}{2}\mathrm{d}A \wedge *\mathrm{d}A - A \wedge *J, \tag{2.1}$$

where A is the $U(1)$ electromagnetic gauge potential one-form and $F = \mathrm{d}A$ is the field strength of it. Also in the units where the speed of light is unity the current one-form in a local coordinate basis $\{\mathrm{d}t, \mathrm{d}x^a\}$ is

$$J = -\rho \mathrm{d}t + \mathbf{J}_a \mathrm{d}x^a, \tag{2.2}$$

where in the temporal component ρ is the charge density and the spatial components $\mathbf{J}_a$ are the current densities. The Lagrangian (2.1) defines a media in which the charge density and the currents are not influenced by the electromagnetic field. The current one-form is predetermined and static that is to say although it acts as a source for the electromagnetic field it does not interact with it dynamically. From (2.1) the inhomogeneous Maxwell equations read

$$d * F = - * J. \tag{2.3}$$

In this section we will consider the coupling of N $U(1)$ gauge field one-forms A^i to the principal sigma model whose target space is a group manifold G. The sigma model Lagrangian can be given as

$$\mathcal{L} = \frac{1}{2}\mathrm{tr}(*\mathrm{d}g^{-1} \wedge \mathrm{d}g). \tag{2.4}$$

Here we take a differentiable map

$$h : M \to G, \tag{2.5}$$

we also consider a representation f of G in $Gl(N, \mathbb{R})$

$$f : G \to Gl(N, \mathbb{R}), \tag{2.6}$$

which may be taken as a differentiable homomorphism. Then the map g can be given as

$$g = f \circ h : M \to Gl(N, \mathbb{R}), \tag{2.7}$$

which is a matrix-valued function on M

$$g(p) = \begin{pmatrix} \varphi^{11}(p) & \varphi^{12}(p) & \cdots \\ \varphi^{21}(p) & \varphi^{22}(p) & \cdots \\ \vdots & \vdots & \vdots \end{pmatrix}, \tag{2.8}$$