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Hidden symmetries in two dimensional field theory

Michael Creutz *

Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA

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Abstract

The bosonization process elegantly shows the equivalence of massless scalar and fermion fields in two space-time dimensions. However, with multiple fermions the technique often obscures global symmetries. Witten's non-Abelian bosonization makes these symmetries explicit, but at the expense of a somewhat complicated bosonic action. Frenkel and Kac have presented an intricate mathematical formalism relating the various approaches. Here, I reduce these arguments to the simplest case of a single massless scalar field. In particular, using only elementary quantum field theory concepts, I expose a hidden $SU(2) \times SU(2)$ chiral symmetry in this trivial theory. I then discuss in what sense this field should be interpreted as a Goldstone boson.

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1. Introduction

A large variety of two dimensional models can be related and often solved via the process of bosonization [1–5]. This process, however, often obscures certain symmetries. For example, the two flavor generalization [6,7] of the Schwinger model [8] in the fermion formulation has an $SU(2) \times SU(2)$ chiral symmetry, but the bosonic solution has one massive and one massless scalar field, both free. In the strong coupling limit the massive particle

* Fax: +1 63 13 445 519.
E-mail address: creutz@bnl.gov

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should become irrelevant, leaving the puzzle of how can a chiral symmetry appear in the trivial theory of only a single free massless scalar.

This question has been discussed in terms of another formulation of bosonization, where the basic fields are elements of a group, and the chiral symmetries involve rotations of these elements. This "non-Abelian" bosonization [9] involves a chiral Lagrangian containing a rather interesting topological term [10,11]. While this formulation keeps the chiral symmetries more transparent, the mapping between the chiral fields and the alternative Abelian bosonization is somewhat obscure. Some of the connections were discussed in a series of papers by Affleck [12,13] and an explicit construction of the connection is given by Amaral and StephanyRuiz [14]. Halpern [7] gives the form for the chiral currents in the multi-flavor Schwinger model.

In this paper, I return to this old topic with further discussion of how a non-Abelian current algebra is hidden in the simplest two dimensional scalar field theory. The construction is well known to the string theory community and is a special case of the general technique of Frenkel and Kac [15–17]. That discussion, however, is placed in a rather formal context; my goal here is to elucidate the surprising consequences more transparently, specializing to the simplest case and using only concepts from elementary quantum field theory.

After a discussion of the connection with Witten's non-Abelian formulation, I turn to some comments on the role and counting of Goldstone bosons. In two dimensions, the definition of a Goldstone boson is subject to some interpretation. On the one hand, infrared fluctuations preclude the matrix valued fields from acquiring a vacuum expectation value. This is the basis of the familiar arguments that spontaneous breaking of a continuous symmetry cannot occur in two dimensions [18]. On the other hand, the chiral charges are rather singular objects, and with a simple cutoff the vacuum is not annihilated by them, even in the limit that the cutoff is removed. The latter is sufficient to require the existence of a massless particle in the spectrum, and forms the basis of one proof of the Goldstone theorem [19]. In this sense a two dimensional field theory can exhibit a Goldstone boson, although it must be free.

Motivated by the simplest case of the strongly coupled two flavor Schwinger model, I will show that the trivial field theory of a free massless boson in one space dimension has a hidden $SU(2)\times SU(2)$ symmetry. In particular, I will construct conserved currents $J^\alpha_{R,\mu}(x)$ and $J^\alpha_{L,\mu}(x)$, where α is an "isospin" index running from 1 to 3, $\mu\in\{0,1\}$ is a Lorentz index, and L,R label left and right-handed parts. The resulting charge densities satisfy the equal time commutation relations:

$$\begin{aligned}
[J^\alpha_{R,0}(x),J^\beta_{R,0}(y)] &= \mathrm{i}\epsilon^{\alpha\beta\gamma}J^\gamma_{R,0}(x)\ \delta(x-y)+A\delta^{\alpha\beta}\partial_x\delta(x-y),\\
[J^\alpha_{L,0}(x),J^\beta_{L,0}(y)] &= \mathrm{i}\epsilon^{\alpha\beta\gamma}J^\gamma_{L,0}(x)\ \delta(x-y)-A\delta^{\alpha\beta}\partial_x\delta(x-y),\\
[J^\alpha_{R,0}(x),J^\beta_{L,0}(y)] &= 0.
\end{aligned} \tag{1}$$

The notation left or right indicates that the corresponding currents are to be constructed from operators involving particles moving to the right or left, respectively. I include here a Schwinger [20] term with coefficient A that will be determined. The Schwinger terms for the two chiralities differ in sign, assuring that they cancel in the commutators of the vector current.

I organize this paper as follows. In Section 2, I make a few remarks on why these two dimensional models may be useful for the understanding of chiral symmetry in four