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Entanglement thresholds for displaying the quantum nature of teleportation

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ABSTRACT

A protocol for transferring an unknown single qubit state evidences quantum features when the average fidelity of the outcomes is, in principle, greater than 2/3. We propose to use the *probabilistic and unambiguous state extraction scheme* as a mechanism to redistribute the fidelity in the outcome of the standard teleportation when the process is performed with an X-state as a noisy quantum channel. We show that the entanglement of the channel is necessary but not sufficient in order for the average fidelity f_X to display quantum features, i.e., we find a threshold C_X for the concurrence of the channel. On the other hand, if the mechanism for redistributing fidelity is successful then we find a filterable outcome with average fidelity $f_{X,0}$ that can be greater than f_X. In addition, we find the threshold concurrence of the channel $C_{X,0}$ in order for the average fidelity $f_{X,0}$ to display quantum features and surprisingly, the threshold concurrence $C_{X,0}$ can be less than C_X. Even more, we find some special cases for which the threshold values become zero.

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1. Introduction

Deterministic teleportation demands three important ingredients: a maximally entangled state as a quantum channel, one-way transfer of classical information, and a pre-existing agreement on the basis of measurement between the two spatially separated laboratories [1–3]. The quantum features of the channel are key resources, since they can be irreversibly affected by the interaction with the surrounding environment. It is well known that different kinds of decoherence, dephasing, and dissipation mechanisms reduce purity and entanglement of the channel. In particular, it has been shown that in different physical systems, an initial Bell state evolves into a stationary X-state when it undergoes non-Markovian interaction mechanisms [4–15]. Therefore, an X-state is a good candidate to be considered as a real noisy quantum channel for implementing quantum protocols. In this context much effort has been devoted to engineer those undesired effects in order to minimize the degrading effects [16–30]. Similarly, a great interest has emerged in the scientific community to characterize the physical bounds and the kind of states, which, despite their mixture degree, are able to manifest quantum features. In this context N. Gisin found the value $2/3$ as the upper bound for the fidelity when teleportation is performed with a particular separable state [31]. S. Popescu shows that, in general, an average fidelity larger than $2/3$ appears when the channel has non-classical features [32]. A relation between the maximal fidelity of teleportation and the maximal singlet fraction attainable by means of local quantum and classical communication action has been found [33–35]. Additionally, the efficiency of teleportation of a continuous-variable state has been studied for the quantum channel of a two-mode squeezed [36–39].

On the other hand, teleportation with a pure partially entangled state can be performed probabilistically. In this case, the original teleportation scheme [1–3] is complemented with an unambiguous state discrimination scheme [40,41] or with an unambiguous state extraction scheme [42–44]. Both schemes have the same probability of achieving successful teleportation, i.e., an outcome with fidelity 1. Recently, an interesting proposal shows a probabilistic scheme without losing the quantum information to be teleported [45]. Experimentally, it is well known that the Rome scheme [46], which has been implemented by using a technique for real-time information transfer, reports an average fidelity beyond the limit of $2/3$ [47]. It is noteworthy that there are a number of interesting proposals for experimental teleportation, for instance Refs. [2,48].

In this paper we show that the unambiguous state extraction (USE) scheme can be used to redistribute the average fidelity in the outcome states when teleportation is performed with an X-state. Additionally, we show that, in both processes, with and without USE, there is a competition between entanglement which introduces quantum features and the dissipation mechanism which decreases the population inside the principal subspace of the quantum channel. Thus, we find threshold values of the concurrence of the channel in order for the quantum features of the processes to be manifested in the average fidelity. The paper is organized as follows: In Section 2 we describe briefly the object of analysis of this work, the average fidelity. In Section 3, we review the teleportation protocol with a pure quantum channel in order to characterize its main aspects. In Section 4, we present a detailed analysis of teleportation carried out with an X-state as the quantum channel. Finally, in Section 5 we summarize our principal results.

2. Average fidelity

Uhlmann's expression for transition probability $F = (Tr\sqrt{\sqrt{\rho}\varrho\sqrt{\rho}})^2$ was proposed as a well defined fidelity of a quantum state ρ with respect to another quantum state ϱ, or vice versa [49–51]. This fidelity accounts for how close or similar the states ϱ and ρ are; for instance, if $\rho = \varrho$ then $F = 1$, otherwise if $\rho \neq \varrho$ then $0 \leq F < 1$. When the states belong to a two-dimensional Hilbert space, R. Jozsa found that $F = Tr\,(\rho\varrho) + 2\sqrt{\det(\rho)\det(\varrho)}$. In this case the identity $\det(\rho) = (1 - Tr\rho^2)/2$ can be applied in order to obtain

$$F = Tr\,(\rho\varrho) + \sqrt{(1 - Tr\rho^2)(1 - Tr\varrho^2)}.$$

If at least one of the two states is pure, say $\rho = |\psi\rangle\langle\psi|$, then $F = \langle\psi|\varrho|\psi\rangle$. In quantum information theory, the challenge is to transmit an unknown pure state $|\psi\rangle$ with the highest fidelity possible via