Annals of Physics 371 (2016) 254–286

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Phase transitions in hexagonal, graphene-like lattice sheets and nanotubes under the influence of external conditions

D. Ebert[a], K.G. Klimenko[b], P.B. Kolmakov[c,*], V.Ch. Zhukovsky[c]

[a] Institute of Physics, Humboldt-University Berlin, 12489 Berlin, Germany

[b] State Research Center of Russian Federation—Institute for High Energy Physics, NRC "Kurchatov Institute", 142281, Protvino, Moscow Region, Russia

[c] Faculty of Physics, Moscow State University, 119991, Moscow, Russia

ARTICLE INFO

Article history:
Received 4 November 2015
Accepted 5 May 2016
Available online 18 May 2016

Keywords:
Effective Lagrangian
Phase transitions
Carbon nanotubes
Zeeman effect
Aharonov–Bohm effect

ABSTRACT

In this paper we consider a class of (2+1)D schematic models with four-fermion interactions that are effectively used in studying condensed-matter systems with planar crystal structure, and especially graphene. Symmetry breaking in these models occurs due to a possible appearance of condensates. Special attention is paid to the symmetry properties of the appearing condensates in the framework of discrete chiral and $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$ transformations. Moreover, boundary conditions corresponding to carbon nanotubes are considered and their relations with the effect of an applied external magnetic field are studied. To this end we calculated the effective potential for the nanotube model including effects of finite temperature, density and an external magnetic field. As an illustration we made numerical calculations of the chiral symmetry properties in a simpler Gross–Neveu model with only one condensate taken into account. We also investigated the phase structure of the nanotube model under the influence of the Aharonov–Bohm effect and demonstrated that there is a nontrivial relation between the magnitude of the Aharonov–Bohm phase,

* Corresponding author.
E-mail address: pavel.b.kolmakov@yandex.ru (P.B. Kolmakov).

http://dx.doi.org/10.1016/j.aop.2016.05.001

compactification of the spatial dimension and thermal restoration of the originally broken chiral symmetry.

1. Introduction

It is a well-known fact that relativistic quantum field theory provides a powerful tool for the description of low-energy excitations in condensed-matter physics [1]. Examples are the field theoretic description of low-energy electron states in polymers [2–4] or the recent quasirelativistic treatment of electrons in planar systems like graphene, a single layer of graphite [5]. Recall that in the case of graphene, the original nonrelativistic tight-binding model for electrons on a hexagonal "honeycomb" lattice admits a low-momentum expansion around the two inequivalent "Dirac points", the corners (valleys) of the first Brillouin zone, which leads to a linear dispersion law for low-energy fermion excitations, closely resembling that of massless relativistic Dirac fermions [5,6]. Combining the two valley degrees of freedom with the two sublattice (pseudospin) degrees of freedom of electrons of carbon atoms, leads in a natural way to a reducible four-component Dirac spinor description in $D = (2+1)$ dimensions. It is just this property which allows for the introduction of a chiral γ_5-matrix and the use of a chiral (Weyl) representation of Dirac matrices [7]. In the continuum limit, the free Dirac Lagrangian of graphene develops an emergent chiral "valley-sublattice" $U(2)_{vs}$ symmetry, which, when considering "multilayer" graphene with N_f flavors, is further enlarged to a chiral $U(2N_f)$ symmetry. There arises then the important question, whether the inclusion of fermion interactions can lead to a dynamical breakdown of chiral symmetry with an associated dynamical fermion mass generation and a "semimetal–insulator" phase transition.

The phenomenon of a dynamical generation of a fermion mass on the basis of a generic four-fermion interaction is well-known for strong interactions since the time, when Nambu and Jona-Lasinio (NJL) [8] generalized the BCS-Bogoliubov theory [9,10] of superconductivity to a relativistic model with dynamical breaking of a continuous γ_5-symmetry. Later on, QCD-motivated NJL-type of models were shown to successfully describe the low-energy meson spectrum of quantum chromodynamics (QCD) [11]. Similar types of four-fermion models with a discrete γ_5-symmetry have also been considered in lower dimensions $D = (1+1)$ by Gross and Neveu (GN) [12], where the four-fermion theory is renormalizable and asymptotic free, or for $D = (2+1)$ in Refs. [13,14]. In the latter case, the model is perturbatively nonrenormalizable but becomes renormalizable in the $1/N_f$ expansion [15].

Generally, four-fermion models provide a useful effective low-energy description of an underlying relativistic fundamental theory. This fact makes it further interesting to investigate their modifications under the influence of external conditions, such as temperature, chemical potential, external magnetic fields etc. [16–19]. Note, on the other hand, that in condensed-matter physics such models are meant to be effective from the very beginning. Non-renormalizability makes here no additional problem due to the natural cutoff in the ultraviolet momentum region provided by the finite spacing between the elements of the polymer lattice. Obviously, one may expect that local four-fermion interactions play also an important role for the generation of a dynamical mass gap and quantum phase transitions in graphene [20–22]. Let us refer in this context also to the interesting investigations based on Schwinger–Dyson equations [23,24], renormalization group flow equations [20] and functional renormalization group methods [25,26].

The main aim of this paper is to continue investigations based on approximative local four-fermion interactions of fermions in a graphene-like hexagonal lattice. In particular, we shall apply the method of the effective potential and the mean-field approach to describe fermionic quasiparticles and excitonic bound states for graphene-like lattice sheets and nanotubes. Moreover, we shall investigate phase transitions under external conditions like temperature, chemical potential and Aharonov–Bohm (AB) magnetic fields [27].