Annals of Physics 371 (2016) 296–312

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Decoherence of odd compass states in the phase-sensitive amplifying/dissipating environment

V.V. Dodonov [a,*], C. Valverde [b,c], L.S. Souza [b], B. Baseia [d,e]

[a] Instituto de Física, Universidade de Brasília, Caixa Postal 04455, 70919-970 Brasília, DF, Brazil
[b] Unidade de Ciências Exatas e Tecnológicas, Universidade Estadual de Goiás, BR 153, km 98, 75001-970 Anápolis, GO, Brazil
[c] Universidade Paulista, BR 153, km 7, 74845-090 Goiânia, GO, Brazil
[d] Instituto de Física, Universidade Federal de Goiás, 74.690-900 Goiânia, GO, Brazil
[e] Departamento de Física, Universidade Federal da Paraíba, 58.051-970 João Pessoa, PB, Brazil

ARTICLE INFO

Article history:
Received 11 March 2016
Accepted 28 April 2016
Available online 6 May 2016

Keywords:
Quantum degenerate parametric amplifier
Phase-sensitive reservoirs
Quantum–classical transition
Wigner function negativity
Fokker–Planck equation
Coherent states superpositions

ABSTRACT

We study the evolution of odd compass states (specific superpositions of four coherent states), governed by the standard master equation with phase-sensitive amplifying/attenuating terms, in the presence of a Hamiltonian describing a parametric degenerate linear amplifier. Explicit expressions for the time-dependent Wigner function are obtained. The time of disappearance of the so called "sub-Planck structures" is calculated using the negative value of the Wigner function at the origin of phase space. It is shown that this value rapidly decreases during a short "conventional interference degradation time" (CIDT), which is inversely proportional to the size of quantum superposition, provided the anti-Hermitian terms in the master equation are of the same order (or stronger) as the Hermitian ones (governing the parametric amplification). The CIDT is compared with the final positivization time (FPT), when the Wigner function becomes positive. It appears that the FPT does not depend on the size of superpositions, moreover, it can be much bigger in the amplifying media than in the attenuating ones.

* Corresponding author.
E-mail address: vdodonov@fis.unb.br (V.V. Dodonov).

http://dx.doi.org/10.1016/j.aop.2016.04.019

Paradoxically, strengthening the Hamiltonian part results in decreasing the CIDT, so that the CIDT almost does not depend on the size of superpositions in the asymptotical case of very weak reservoir coupling. We also analyze the evolution of the Mandel factor, showing that for some sets of parameters this factor remains significantly negative, even when the Wigner function becomes positive.

1. Introduction

Various superpositions of coherent states (introduced in quantum optics in [1,2]) have been studied from different points of view in numerous papers for almost five decades. In particular, much attention was paid to different superpositions of N coherent states of the form $\sum_{k=0}^{N-1} c_k|\alpha \exp(2\pi ik/N)\rangle$. The first examples were considered in [3,4], where the coefficients c_k were selected in such a way that the photon statistics remained Poissonian, like in the original coherent states. All such states are eigenstates of the Nth power of the annihilation operator $\hat{a}$, and choosing suitable values of c_k one can construct N orthogonal superpositions. The simplest case of $N = 2$ was considered for the first time in [5], where the so called *even/odd coherent states* $|\alpha\rangle_\pm = A_\pm (|\alpha\rangle \pm |-\alpha\rangle)$ were introduced ($A_\pm$ is the normalization factor, whose explicit form is not important for our purposes). The states $|\alpha\rangle_+$ and $|\alpha\rangle_-$ are two orthogonal eigenstates of the squared annihilation operator $\hat{a}^2$. The photon statistics in the even states is super-Poissonian, whereas it is sub-Poissonian for the odd states. Properties of states $|\alpha\rangle_\pm$, their generalizations, schemes of generation and evolution under various conditions were studied by many authors, cited, e.g., in reviews [6–8]. For more recent publications see, e.g., [9–21]. More general *multi-photon states* for $N > 2$ were studied, e.g., in [22–28].

Using *four* different coherent states $|\alpha_j\rangle$ one can construct a lot of different superpositions, multiplying each state $|\alpha_j\rangle$ by different complex amplitudes c_j. Interesting specific families of states arise if one combines two even/odd coherent states $|\alpha\rangle_\pm$ and $|i\alpha\rangle_\pm$ with different coefficients. The first such combination $|\alpha\rangle_+ - |i\alpha\rangle_-$ was considered in [29]. It is one of eigenstates of operator $\hat{a}^4$. The combinations $|i\alpha\rangle_+ \pm \exp(\pm i\pi/4)|\alpha\rangle_-$ were studied in [23], whereas the superposition $|\alpha\rangle_+ + |i\alpha\rangle_+$ (called *orthogonal-even state*) was introduced in [30]. The sum of states $|\alpha\rangle_- + \frac{1}{2}|i\alpha\rangle_+$ appeared in [31]. All four orthogonal eigenstates of operator $\hat{a}^4$ in the form of combinations of $|\alpha\rangle_\pm$ and $|i\alpha\rangle_\pm$ were introduced under the name *four-photon states* in [32]. These states (studied also in [33,34]) can be considered as special cases of a more general family of "coherent states on a circle", introduced in [35]. For brevity, we shall call them *circular states*. This name is frequently used nowadays for superpositions of N coherent states of the form

$$|\alpha; N; m\rangle = A\sum_{k=0}^{N-1} \exp(-2\pi ikm/N)|\alpha \exp(2\pi ik/N)\rangle, \tag{1}$$

where A is the normalization factor. The states (1) can be represented also as infinite superpositions of the Fock states $|m+Nj\rangle$ (with $j = 0, 1 \dots, \infty$ and $m = 0, 1, \dots, N-1$), therefore they are orthogonal. Such states were studied in detail, e.g., in [36–43]. More general superpositions, arising in the process of evolution of an initial coherent state in Kerr media, were considered in [44] (the special case of $N = 2$ was studied earlier in [29,45]); see also the review [46].

The name *compass states* was coined in [47] for the special case of four-photon states in the form $|\alpha\rangle_+ + |i\alpha\rangle_+$ (i.e., the same state as in [30]), due to the profile of its Wigner function. The emphasis in [47] was made on the *sub-Planck structures* in the phase space. Further studies in this direction were performed in Refs. [48–55] (the name "four-headed cat state" was used in [55]).