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Correlation energy for elementary bosons: Physics of the singularity

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ABSTRACT

We propose a compact perturbative approach that reveals the physical origin of the singularity occurring in the density dependence of correlation energy: like fermions, elementary bosons have a singular correlation energy which comes from the accumulation, through Feynman "bubble" diagrams, of the same non-zero momentum transfer excitations from the free particle ground state, that is, the Fermi sea for fermions and the Bose–Einstein condensate for bosons. This understanding paves the way toward deriving the correlation energy of composite bosons like atomic dimers and semiconductor excitons, by suggesting Shiva diagrams that have similarity with Feynman "bubble" diagrams, the previous elementary boson approaches, which hide this physics, being inappropriate to do so.

1. Introduction

The energy of N interacting elementary bosons has been studied in the late 1950s by Brueckner and Sawada [1], and by Lee, Huang and Yang [2,3] through a mean-field procedure which transforms the two-body Hamiltonian into a quadratic operator easy to diagonalize by using a Bogoliubov-like

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transformation. Their most striking result is that the N-boson correlation energy is singular with a dependence on density $n = N/L^3$ in $n^{3/2}$ instead of n^2. More precisely, the ground-state energy of N interacting bosons reads as

$$\frac{\mathcal{E}_N}{N} \simeq \frac{2\pi}{ma^2}\left[na^3 + \frac{128}{15\sqrt{\pi}}(na^3)^{3/2}\right] \tag{1}$$

where m is the boson mass and a is the scattering length of the boson–boson potential at hand. Actually, the above result with the scattering length appearing also in the $n^{3/2}$ term has been obtained by Lee, Huang and Yang, but not by Brueckner and Sawada. The reason is that the former authors use a pseudo-potential which reads in terms of a; so, they do not have to bother about generating the scattering length in the correlation term. The drawback of these two previous approaches is that they rely on a mean-field approximation which completely hides the physics of the correlation energy singularity. A more elaborate field-theory procedure has been later proposed [4]. It provides a better control of the performed approximations, but through a more complicated procedure that still hides the physical origin of the singularity.

The purpose of this paper is to reveal that, in spite of the different quantum nature of the particles and the different singular dependence in the correlation energy of bosons and fermions [5], namely $(n^{3/2})$ and $(\ln n)$, the physics producing these singular dependences is just the same: the accumulation of the same non-zero momentum transfer excitations from the free particle ground state, that is, the $\mathbf{k} = \mathbf{0}$ boson condensate or the $0 \leqslant |\mathbf{k}| \leqslant k_F$ Fermi sea. This understanding paves the way toward deriving the correlation energy of N composite bosons made of fermion pairs by suggesting the appropriate set of Shiva diagrams [6] that have similarity with the Feynman "bubble" diagrams leading to the singular correlation energy of elementary bosons. Composite bosons of major present interest are atomic dimers [7–9] made of different species of cold fermionic atoms [10–12], semiconductor excitons [13–15] made of electrons and holes, and polaritons [16–19] which are linear combination of photons and excitons. Cold atoms [20,21] and polaritons [22] have been used as a testbed for low-energy (Bogoliubov) excitations in the mean-field framework of elementary bosons. Whether composite bosons defy this mean-field description because of their composite nature remains under debate in spite of the fact that, because of the fermion indistinguishability, one cannot construct an effective potential between composite bosons that is valid beyond first order in interaction [6]. As a direct consequence, elementary boson approaches based on the existence of a boson–boson potential cannot be duplicated for composite boson systems.

To show the analogy between the correlation energies of elementary bosons and elementary fermions, we propose a compact perturbative approach to the energy of N quantum particles that allows catching the physics of the singular interaction processes. Once selected and summed up, these singular processes lead to the N-boson energy given in Eq. (1). Unlike previous methods, the perturbative approach that we here propose can be directly extended to composite bosons which interact not only through the fermion–fermion interactions between their fermionic components, but also through fermion exchanges.

The paper is organized as follows:

• We first give some general arguments for understanding what should be and what really is the density dependence of the correlation energies for N elementary bosons and N elementary fermions, in order to understand why these density dependences end up being different, albeit produced by the same physical processes.

• Next, we propose a compact perturbative approach to derive the N-boson energy, which allows catching the physics of its various terms in a transparent way. We also provide the key commutators which enable us to calculate these terms easily.

• We then recover the N-boson energy given in Eq. (1) through the explicit summations of the ladder diagrams associated with the scattering length, and of the bubble diagrams associated with the correlation energy singularity. We also discuss the required cancellation of overextensive contributions that come from disconnected diagrams, as standard in perturbative expansion.

• We conclude with the state-of-the-art for composite boson systems and the fundamental problem raised by the Pauli exclusion principle between the fermionic components of composite quantum particles.