Annals of Physics 367 (2016) 96–124

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

Floquet–Magnus theory and generic transient dynamics in periodically driven many-body quantum systems

Tomotaka Kuwahara [a,b,*], Takashi Mori [a], Keiji Saito [c]

[a] *Department of Physics, Graduate School of Science, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan*

[b] *WPI, Advanced Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan*

[c] *Department of Physics, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama, 223-8522, Japan*

HIGHLIGHTS

- A general framework to describe transient dynamics for periodically driven systems.
- The theory is applicable to generic quantum many-body systems including long-range interacting systems.
- Physical meaning of the truncation of the Floquet–Magnus expansion is rigorously established.
- New mechanism of the prethermalization is proposed.
- Revealing an experimental time-scale for which non-trivial dynamical phenomena can be reliably observed.

ARTICLE INFO

Article history:
Received 2 November 2015
Accepted 17 January 2016
Available online 22 January 2016

Keywords:
Periodically driven system
Generic few-body Hamiltonian
Transient dynamics
Floquet–Magnus expansion
Prethermalization
Energy absorption

ABSTRACT

This work explores a fundamental dynamical structure for a wide range of many-body quantum systems under periodic driving. Generically, in the thermodynamic limit, such systems are known to heat up to infinite temperature states in the long-time limit irrespective of dynamical details, which kills all the specific properties of the system. In the present study, instead of considering infinitely long-time scale, we aim to provide a general framework to understand the *long but finite time* behavior, namely the transient dynamics. In our analysis, we focus on the Floquet–Magnus (FM) expansion that gives a formal expression of the effective Hamiltonian on the system. Although in general the full series expansion is not convergent in the thermodynamics limit, we give a clear relationship between the FM expansion and the transient dynamics. More

* Corresponding author at: WPI, Advanced Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan.
E-mail address: tomotaka.phys@gmail.com (T. Kuwahara).

http://dx.doi.org/10.1016/j.aop.2016.01.012

precisely, we rigorously show that a truncated version of the FM expansion accurately describes the exact dynamics for a certain time-scale. Our theory reveals an experimental time-scale for which non-trivial dynamical phenomena can be reliably observed. We discuss several dynamical phenomena, such as the effect of small integrability breaking, efficient numerical simulation of periodically driven systems, dynamical localization and thermalization. Especially on thermalization, we discuss a generic scenario on the prethermalization phenomenon in periodically driven systems.

1. Introduction

1.1. Physical background

Periodically driven quantum dynamics has recently attracted much attention in experimental as well as theoretical studies [1–5] as it offers a promising way for exploring novel quantum phenomena which would be difficult or impossible to observe otherwise. Although the instantaneous Hamiltonian at each time step is very simple, dynamical behavior can be highly nontrivial. Remarkable dynamical phenomena include dynamical localization [6–10], coherent destruction of tunneling [11,12,4], localization–delocalization transition [13–17], and dynamical phase transitions [18–22]. Moreover, recent experimental development has rapidly opened new possibilities to control quantum systems under periodic driving, e.g., in the context of quantum transport [5,23–25], quantum topological phases [26–32] and detections of the Majorana Fermion [33,34] and the Higgs mode in condensed matter [35].

One of main subjects in driven quantum many-body systems is to understand the thermodynamical structure of steady states. With a few exceptions [10,18–22], recent studies mostly focus on driving simple (often non-interacting particles) Hamiltonians. However, the integrability-breaking terms unavoidably exist in the realistic experimental conditions. When one looks at long-time behavior, even small integrability-breaking terms are relevant to the dynamics and cause significant effects on the final steady states. This provides the motivation to understand the long-time behavior of 'generic' many-body systems under periodic driving. Long-time behaviors in driven non-integrable systems are in general very complicated and cannot be captured with simple techniques such as the rotating-wave approximation and the transfer matrix technique of Landau–Zener transitions [6] etc., and hence one is obliged to rely on numerical calculations.

Recently, true steady states in the long-time limit have been intensively studied using large scale numerical calculations [10,36,16]. In the long-time limit, periodically driven many-body systems are in general expected to heat up to infinite temperature. This is a consequence from the analogy of eigenstate thermalization hypothesis (ETH) in non-driven many-body systems [37]. The ETH implies that each energy eigenstate of Hamiltonian is indistinguishable from the microcanonical ensemble with the same energy. In periodically driven systems, the energy is no longer conserved and hence the extension of ETH to the driven case indicates that the steady state is a state of infinite temperature (i.e., the completely random state). Thus, due to the heating effect, any information reflected from the system is invisible in the *infinite-time scale.*

However, the experiments on many-body quantum systems do not focus on the long-time limit; rather, they are interested in the transient dynamics for the *experimental time scale.* In this time scale, the heating process cannot necessarily occur, and hence a critical task that follows is to clarify the time-scale during which one can observe *transient behavior* that can show nontrivial phenomena. So far, most studies on transient dynamical properties are based on numerical calculations with phenomenological arguments.