Annals of Physics 367 (2016) 199–211

Contents lists available at ScienceDirect

Annals of Physics

journal homepage: www.elsevier.com/locate/aop

# Error correction in short time steps during the application of quantum gates

L.A. de Castro *, R.d.J. Napolitano

*São Carlos Institute of Physics, University of São Paulo, PO Box 369, 13560-970, São Carlos, SP, Brazil*

A R T I C L E  I N F O

*Article history:*
Received 6 July 2015
Accepted 20 January 2016
Available online 28 January 2016

*Keywords:*
Quantum code
Quantum measurement

A B S T R A C T

We propose a modification of the standard quantum error-correction method to enable the correction of errors that occur due to the interaction with a noisy environment during quantum gates without modifying the codification used for memory qubits. Using a perturbation treatment of the noise that allows us to separate it from the ideal evolution of the quantum gate, we demonstrate that in certain cases it is necessary to divide the logical operation in short time steps intercalated by correction procedures. A prescription of how these gates can be constructed is provided, as well as a proof that, even for the cases when the division of the quantum gate in short time steps is not necessary, this method may be advantageous for reducing the total duration of the computation.


## 1. Introduction

Since the beginning of quantum-computation theory, it has been known that decoherence and other forms of external interference in the quantum bits (qubits) pose a difficulty in implementing real quantum computers [1]. Various methods have been devised to cope with these errors, including the quantum error-correction theory, which consists of encoding quantum states of $k$ logical qubits in $n > k$ physical qubits, thus generating a redundancy of information that can be used to identify and correct errors [2]. Many of these codes have since been designed for different kinds of quantum channels.

* Corresponding author.
*E-mail address:* leonardo.castro@usp.br (L.A. de Castro).

http://dx.doi.org/10.1016/j.aop.2016.01.014

It has been shown that quantum codes can be employed in quantum circuits as long as the gates are fault-tolerant [3]. The traditional fault-tolerant methods consider that the errors can be described as happening after the physical gates are applied [4], which is a harmless assumption when dealing with a code capable of correcting any generic error up to a certain number of qubits. In this case, any kind of error that may emerge from the simultaneous interaction of the gate and the environment will still be correctable.

This assumption, nevertheless, does not remain true if we take into account quantum gates that have finite-time durations which are not negligibly small in comparison with the decoherence time [5]. When we let both the errors and the gates occur simultaneously, unexpected new kinds of errors may affect the encoded qubits, which may render the correction procedure impractical if we are using a code that was designed to correct the noise of a specific quantum channel. Given that such channel-adapted codes can be more efficient than generic ones [6], a method of correcting errors that occur during the gate using the same specific code developed for the case of a memory qubit deserves attention.

In this article, we propose an alternative method of correcting errors caused by an external environment occurring during a logical operation that consists of repeatedly applying the quantum error-correction procedure for memory qubits. Employing the analytic theoretical methods of standard quantum coding theory and of the dynamical evolution of a statistical quantum ensemble given by the von-Neumann equation, we lay down the conditions for the repeated error correction to be necessary. We also show how it can be applied to correct a universal set of quantum gates, exemplified in the case of the three-qubit phase-error-correcting code [7].

Special attention is given to the form such gates must be constructed. While fault-tolerant circuits employ transversal gates, which have been proven incapable of constructing a universal set for codes that correct a general one-qubit error [8], our method bypasses such restrictions by employing gates which cannot be factored in a tensor product of individual-qubit operations, being therefore non-transversal. The use of such gates does not cause additional errors with significant probability, but may require some special arrangement, as explained in the body of the article.

This article is structured as follows: in Section 2, we examine the essential mathematics necessary to understand the small-step method. In Section 3, we explain how the method works, and in Section 4 we show how the gates can be implemented within this framework. Section 5 is dedicated to describe how the gates can be implemented by an approximative method when multiple-qubit interactions are prescribed. We conclude in Section 6, presenting some further perspectives of work.

## 2. Mathematical formulation

In the description of a quantum computer prone to errors, we consider that the qubits are an object-system $S$ (to employ the same terminology as [9]) that interacts with an environment $E$, usually modeled as bosonic baths. Even though we are interested only in the reduced density matrix of the qubits $\rho_S(t) = \mathrm{Tr}_E\{\rho_{SE}(t)\}$, we must take into account the whole system and environment, described by the total density operator $\rho_{SE}(t)$, so we can describe a unitary evolution given by the von Neumann equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{SE}(t) = -i\left[H, \rho_{SE}(t)\right],$$

where $H$ is the Hamiltonian operating on both object-system and environment and we chose natural units, so that $\hbar = 1$.

The solution of this equation is found to be

$$\rho_{SE}(t) = U_{SE}(t)\rho_{SE}(0)U_{SE}^{\dagger}(t), \tag{1}$$

where the time-evolution operator $U_{SE}(t)$ is $e^{-iHt}$ if the Hamiltonian is time-independent. This Hamiltonian can always be split in three terms: two that act on the object-system and environment separately ($H_S$ and $H_E$) and one term of interaction that acts on both ($H_{\mathrm{int}}$). The latter must be weak enough for the errors to be rare—otherwise, the recovery of the original state will be