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Geometric optics for a coupling model of electromagnetic and gravitational fields

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The coupling between the electromagnetic and gravitational fields results in "faster than light" photons, and then the first and third laws of geometric optics are invalid in usual spacetime. By introducing an effective spacetime, we find that the wave vector can be casted into null and then it obeys the geodesic equation, the polarization vector is perpendicular to the rays, and the number of photons is conserved. That is to say, the laws of geometric optics are valid for the modified theory in the effective spacetime. We also show that the focusing theorem of light rays for the modified theory in the effective spacetime can be cast into the usual form.

1. Introduction

It is well known that the propagation of light and radio waves are subject to the laws of geometric optics. The fundamental laws of geometric optics are: (1) light rays are null geodesics; (2) the polarization vector is perpendicular to the rays and parallel-propagated along the rays; and (3) the amplitude is governed by an adiabatic invariant which, in the quantum language, states that the number of photons is conserved. The conditions under which these laws hold are defined by conditions on three lengths: (1) the typical reduced wavelength of the waves, $\bar{\lambda}$, as measured in a typical local Lorentz frame; (2) the typical length $\mathfrak{L}$ over which the amplitude, polarization, and

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wavelength of the waves vary, e.g., the radius of curvature of a wavefront; (3) the typical radius of curvature $\mathfrak{R}$ of the spacetime through which the waves propagate. Geometric optics is valid whenever the reduced wavelength is very short compared to each of the other scales. We should note that these laws, regardless of in the flat or curved spacetime, are derived from the usual free Maxwell field.

Recent investigations show that the interaction between the electromagnetic and gravitational fields could be appeared naturally in quantum electrodynamics with the photon effective action originating from one-loop vacuum polarization in curved spacetime [1]. These coupling models of the electromagnetic and gravitational fields are of great interest, since the appearance of cross-terms in the Lagrangian leads to the modifications of the coefficients involving the higher-order derivatives both in the Maxwell and Einstein equations. So the electromagnetic theories containing this coupling term have been studied extensively [1–4]. Drummond and Hathrell [1] argued that the quantum corrections introduce tidal gravitational forces on the photons which alter the characteristics of propagation, so that in some cases photons travel at speeds greater than unity. The one-loop effective action for QED in curved spacetime contains the equivalence principle violating interactions between the electromagnetic field and the spacetime curvature, and these interactions lead to a dependence of the photon velocity on the motion and polarization directions [1,5–10]. By taking the analogy between the eikonal equation in geometric optics and the particle equation of motion, Ahmadi and Nouri-Zonoz [11] investigated the phase structure and the trajectory of the propagating photon semiclassically in the modified theory.

Although the coupling models for the electromagnetic and gravitational fields have been extensively used recently, these models will result in "faster than light" photons. Therefore, for these coupling models, the characteristics of propagation of the light are altered and the laws of geometric optics, in general, are invalid in the usual spacetime. The reason is that the electromagnetic field is described by the modified theory rather than the usual free Maxwell theory now. In this paper, we want to show that, by introducing an effective spacetime, the laws of geometric optics and focusing theorem of light rays are still valid for the modified theory.

The plan of the paper is as follows. In the next section we introduce a model for the interaction between the electromagnetic and gravitational fields, and obtain the motion equations of light by using the geometric optics approximation in the usual spacetime. In section III, by using the effective spacetime introduced in the Appendix, we investigate the laws of geometric optics and the focusing theorem of light rays. We present our conclusions in the last section.

2. Modified theory for couplings between electromagnetic and gravitational fields

To study the interactions between the electromagnetic and gravitational fields, it is natural to consider the couplings between the Maxwell field and the Weyl tensor. Therefore, the electromagnetic theory with the Weyl correction has been studied extensively [12–17]. The action for a toy model of the electromagnetic field coupled to the Weyl tensor can be expressed as [12]

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} - \frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu} - 4\alpha C^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}\right)\right], \tag{2.1}$$

where $F_{\mu\nu} = A_{\nu;\mu} - A_{\mu;\nu}$ is the usual electromagnetic tensor with a vector potential A_μ, α is the coupling constant which has the dimension of the length-squared, and $C_{\mu\nu\rho\sigma}$ is the Weyl tensor defined by

$$C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - (g_{\mu[\rho}R_{\sigma]\nu} - g_{\nu[\rho}R_{\sigma]\mu}) + \frac{1}{3}Rg_{\mu[\rho}g_{\sigma]\nu}, \tag{2.2}$$

where $g_{\mu\nu}$ represents the usual spacetime and the brackets around indices refer to the antisymmetric part. Varying the action (2.1) with respect to A_μ, we obtain

$$\nabla_\mu(F^{\mu\nu} - 4\alpha C^{\mu\nu\rho\sigma}F_{\rho\sigma}) = 0. \tag{2.3}$$

Under the geometric optics assumption, we can set

$$A^\mu = (a^\mu + \varepsilon b^\mu + \varepsilon^2 c^\mu + \cdots\cdots)e^{i\theta}, \tag{2.4}$$